

Image pyramids

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The Laplacian Pyramid as a Compact Image Code

PETER J. BURT, MEMBER, IEEE, AND EDWARD H. ADelson

Abstract—We describe a technique for image encoding in which local operators of many scales but identical shape serve as the basis functions. The representation differs from established techniques in that the code elements are localized in spatial frequency as well as in space.

Pixel-to-pixel correlations are first removed by subtracting a low-pass filtered copy of the image from the image itself. The result is a net data compression since the difference, or error, image has low variance and entropy, and the low-pass filtered image may be represented at reduced sample density. Further data compression is achieved by spanning the difference image. These steps are then repeated to compress the low-pass image, iteration of the process at appropriately expanded scales generates a pyramidal data structure.

The encoding process is equivalent to sampling the image with Laplacian operators of many scales. Thus, the code tends to enhance useful image features. A further advantage of the present code is that it is well suited for many image analysis tasks as well as for image compression. Fast algorithms are described for coding and decoding.

INTRODUCTION

A COMMON characteristic of images is that neighboring pixels are highly correlated. To represent the image directly in terms of the pixel values is therefore inefficient; most of the encoded information is redundant. The first task in designing an efficient, compressed code is to find a representation which, in effect, decorrelates the image pixels. This has been achieved through predictive and through transform techniques [cf. [9], [10] for recent reviews].

In predictive coding, pixels are encoded sequentially in a raster format. However, prior to encoding each pixel, its value is predicted from previously coded pixels in the same and preceding raster lines. The predicted pixel value, which represents redundant information, is subtracted from the actual pixel value, and only the difference, or prediction error, is encoded. Since only previously encoded pixels are used in predicting each pixel's value, this process is said to be causal. Restrictions to causal prediction facilitate decoding; to decode a given pixel, its predicted value is recomputed from already decoded neighboring pixels, and added to the stored prediction error.

Noncausal prediction, based on a symmetric neighborhood centered at each pixel, should yield more accurate prediction and, hence, greater data compression. However, this approach

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does not permit simple sequential coding. Noncausal approaches to image coding typically involve image transforms, or the solution to large sets of simultaneous equations. Rather than encoding pixels sequentially, such techniques encode them all at once, or by blocks.

Both predictive and transform techniques have advantages. The former is relatively simple to implement and is readily adapted to local image characteristics. The latter generally provides greater data compression, but at the expense of considerably greater computation.

Here we shall describe a new technique for removing image correlation which combines features of predictive and transform methods. The technique is noncausal, yet computations are relatively simple and local.

The predicted value for each pixel is computed as a local weighted average, using a unimodal Gaussian-like (or related unimodal) weighting function centered on the pixel itself. The predicted values for all pixels are first obtained by convolving this weighting function with the image. The result is a low-pass filtered image which is then subtracted from the original.

Let $g(i,j)$ be the original image, and $g_c(i,j)$ be the result of applying an appropriate low-pass filter to g . The prediction error $L(i,j)$ is then given by

$$L(i,j) = g(i,j) - g_c(i,j)$$

Rather than encode g , we encode L and g_c . This results in a net data compression because a) L is largely decorrelated, and so may be represented pixel by pixel with many fewer bits than g , and b) g_c is low-pass filtered, and so may be encoded at a reduced sample rate.

Further data compression is achieved by iterating this process. The reduced image g_c is itself low-pass filtered to yield g_c , and a second error image is obtained: $L_c(i,j) = g_c(i,j) - g_{cc}(i,j)$. By repeating these steps several times we obtain a sequence of two-dimensional arrays $L_0, L_1, L_2, \dots, L_n$. In our implementation each is smaller than the predecessor by a scale factor of 1/2 due to reduced sample density. If we now imagine these arrays stacked one above another, the result is a superior pyramidal data structure. The value at each node in the pyramid represents the difference between two Gaussian-like or related functions convolved with the original image. The difference between these two functions is similar to the "Laplacian" operators commonly used in image enhancement [13]. Thus, we refer to the proposed compressed image representation as the Laplacian-pyramid code.

The coding scheme outlined above will be practical only if rapid filtering computations can be performed with an efficient algorithm. A suitable fast algorithm has recently been developed [2] and will be described in the next section.

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From Wikipedia

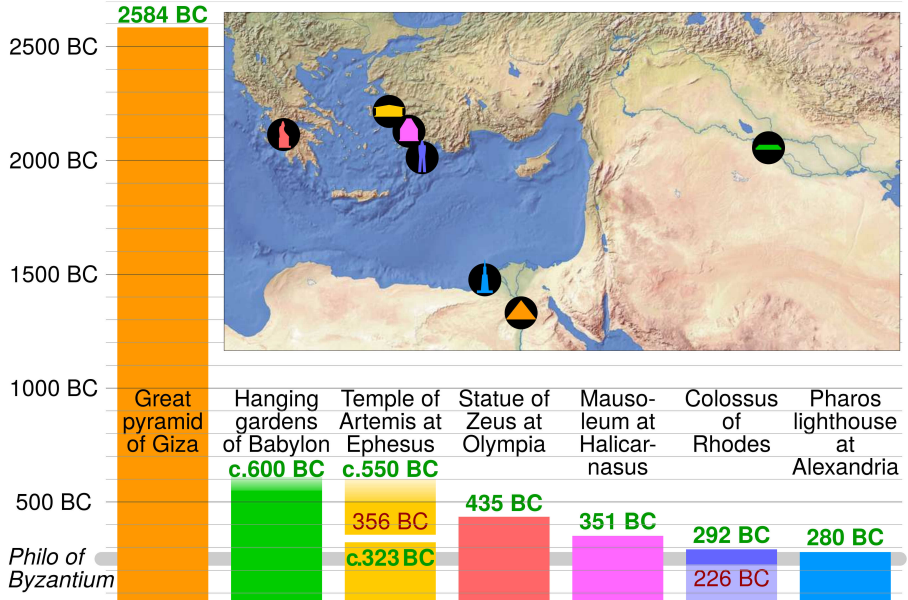
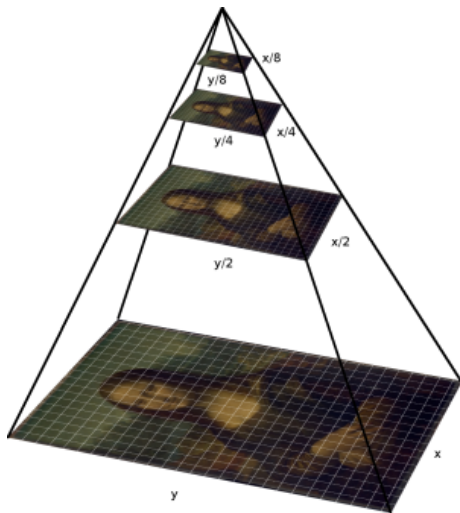


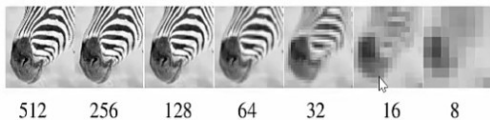
Image pyramid

- Very useful for representing images
- Pyramid is built as multiple resolution approximations of a same image
- Each level in the pyramid is $1/4$ of the size of the previous level
- Lowest level has highest resolution
- Highest level has lowest resolution



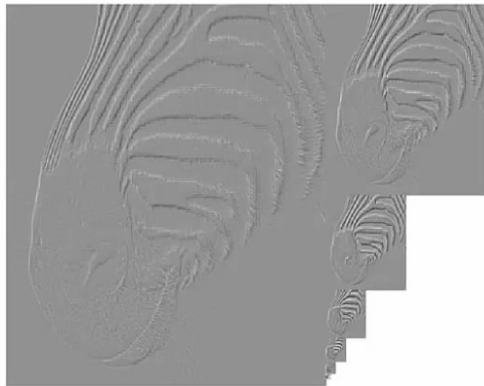
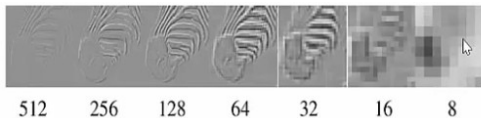
<https://www.pyimagesearch.com/2015/03/16/image-pyramids-with-python-and-opencv/>

Gaussian pyramid



Source: Forsyth

Laplacian pyramid



Source: Forsyth

Things to learn today

- Gaussian and Laplacian pyramid
 - Reduce
 - Expand
- Applications of Laplacian pyramid
 - Image compression
 - Image compositing

Reduce operation

$$g_l = \text{REDUCE}[g_{l-1}]$$

$$\underbrace{g_l(i, j)}_{l\text{-level}} = \sum_{m=-2}^2 \sum_{n=-2}^2 \tilde{w}(m, n) \underbrace{g_{l-1}(2i - m, 2j - n)}_{l-1\text{-level}}$$

Reduce operation

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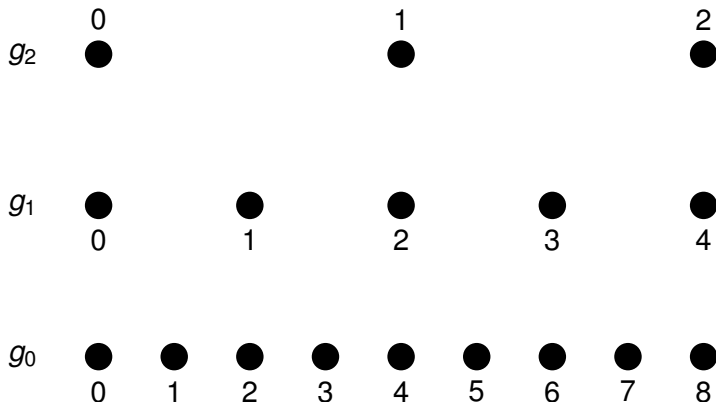
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Remark

Note that it is different from convolution that we skip every one sample per dimension for the input

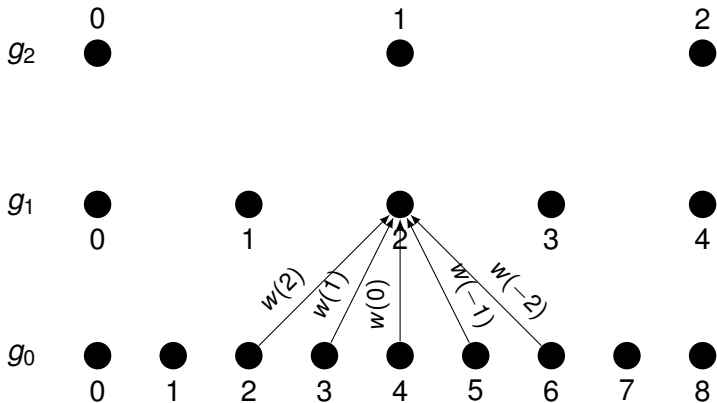
1-D case

$$g_l(i) = \sum_{m=-2}^2 w(m)g_{l-1}(2i - m)$$



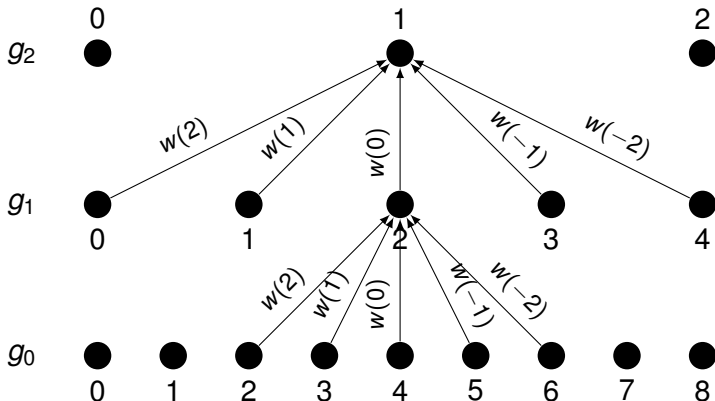
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Expand operation

$$\hat{g}_l = EXPAND[\hat{g}_{l-1}]$$

$$\hat{g}_l(i, j) = \sum_{n=-2}^2 \sum_{m=-2}^2 \tilde{w}(m, n) \hat{g}_{l-1} \left(\frac{i-n}{2}, \frac{j-m}{2} \right)$$

Remark

EXPAND is approximately the inverse operation of REDUCE

1-D case

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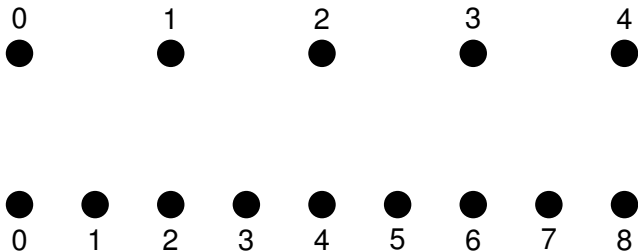
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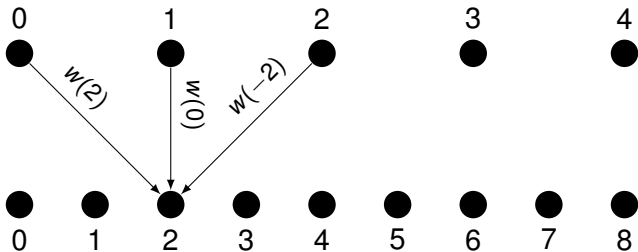


Remark

Note that since we have discard half of the coefficients during expansion, we should double the weights $w(m)$ comparing to those during reduction

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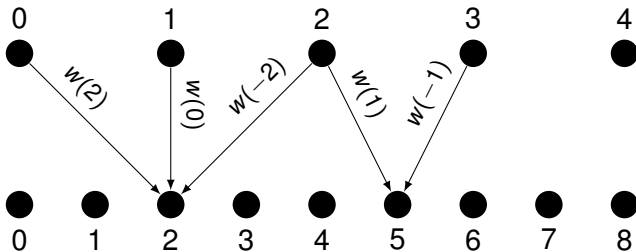


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Design properties of convolution mask

Separable

- That is, $\tilde{w}(m, n) = w(m)w(n)$
- We can save computational cost (from N^2 to $2N$)

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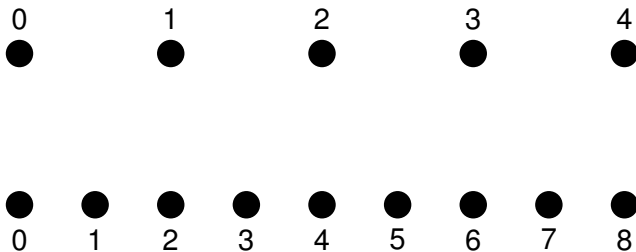
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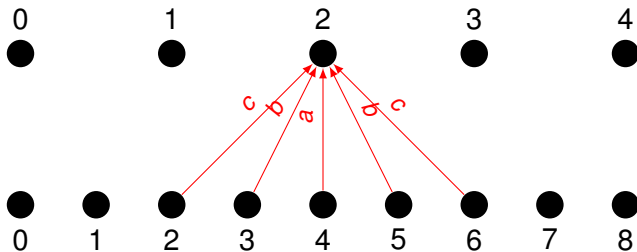
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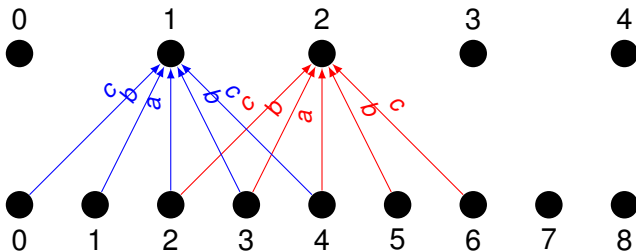
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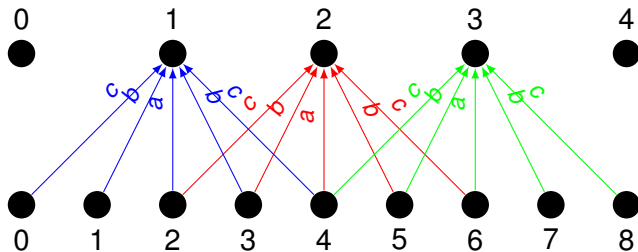
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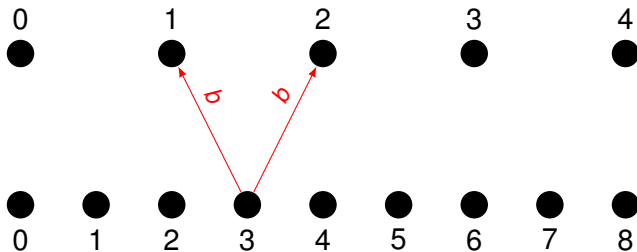
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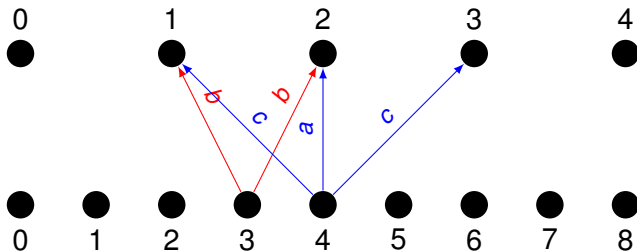
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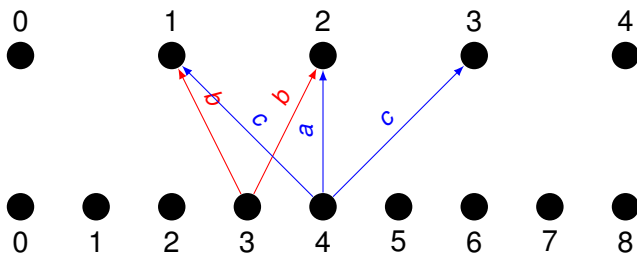
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Normalized

- Sum of mask should be 1
- Thus, $a + 2b + 2c = 1$

“Unbiased”

- All nodes at a given level should contribute the same to nodes at the next level ($2b = 2c + a$)



Mask parameters

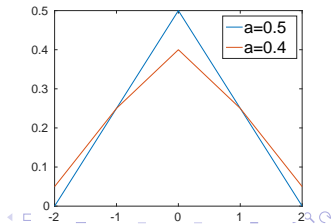
- From the previous slide,

$$\begin{cases} a + 2c = 2b \\ a + 2b + 2c = 1 \end{cases} \Rightarrow 4b = 1 \Rightarrow b = \frac{1}{4}$$

- Substitute $b = \frac{1}{4}$ back to the equations, we have

$$\begin{cases} b = \frac{1}{4} \\ c = \frac{1}{4} - \frac{a}{2} \end{cases}$$

- $a = 0.4 \approx$ Gaussian, $a = 0.5 \approx$ triangular



Laplacian pyramids

- Similar to edge detected images
- Most pixels are zero
- Can be used for image compression

$$L_1 = g_1 - \text{EXPAND}[g_2]$$

$$L_2 = g_2 - \text{EXPAND}[g_3]$$

$$L_3 = g_3 - \text{EXPAND}[g_4]$$

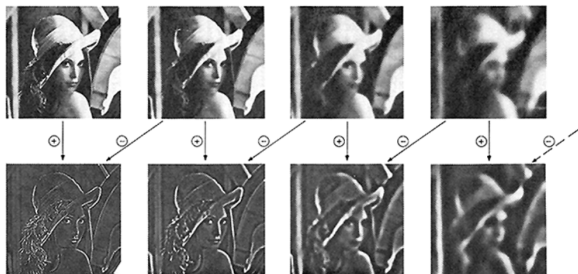


Image encoding

- Compute Gaussian pyramid

g_1, g_2, g_3, g_4

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- Compute Gaussian pyramid

$$g_1, g_2, g_3, g_4$$

- Compute Laplacian pyramid

$$L_1 = g_1 - \text{EXPAND}[g_2]$$

$$L_2 = g_2 - \text{EXPAND}[g_3]$$

$$L_3 = g_3 - \text{EXPAND}[g_4]$$

$$L_4 = g_4$$

Image encoding

- Compute Gaussian pyramid

$$g_1, g_2, g_3, g_4$$

- Compute Laplacian pyramid

$$L_1 = g_1 - \text{EXPAND}[g_2]$$

$$L_2 = g_2 - \text{EXPAND}[g_3]$$

$$L_3 = g_3 - \text{EXPAND}[g_4]$$

$$L_4 = g_4$$

- Quantize and code Laplacian pyramid (e.g., by Huffmann coding)

Image encoding

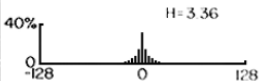
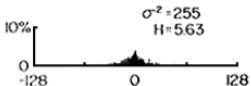
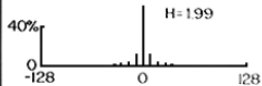
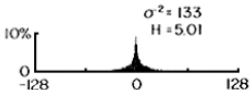
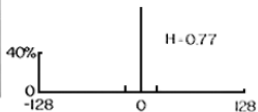
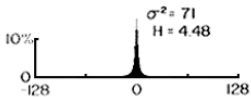
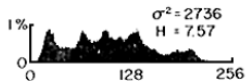


Image decoding

- Decode Laplacian pyramid

$$L_1, L_2, L_3, L_4$$

Image decoding

- Decode Laplacian pyramid

$$L_1, L_2, L_3, L_4$$

- Compute Gaussian pyramid from Laplacian pyramid

$$g_4 = L_4$$

$$g_3 = L_3 + \text{EXPAND}[g_4]$$

$$g_2 = L_2 + \text{EXPAND}[g_3]$$

$$g_1 = L_1 + \text{EXPAND}[g_2]$$

Image decoding

- Decode Laplacian pyramid

$$L_1, L_2, L_3, L_4$$

- Compute Gaussian pyramid from Laplacian pyramid

$$g_4 = L_4$$

$$g_3 = L_3 + \text{EXPAND}[g_4]$$

$$g_2 = L_2 + \text{EXPAND}[g_3]$$

$$g_1 = L_1 + \text{EXPAND}[g_2]$$

- g_1 is the reconstructed image

Compression result



(a)



(b)



(c)

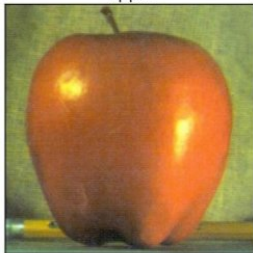


(d)

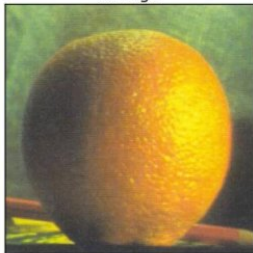
Fig 8. Examples of image data compression using the Laplacian Pyramid code. (a) and (c) give the original "Lady" and "Walter" images, while (b) and (d) give their encoded versions of the data rates are 1.58 and 0.73 bits/pixel for "Lady" and "Walter," respectively. The corresponding mean square errors were 0.88 percent and 0.43 percent, respectively.

Image compositing

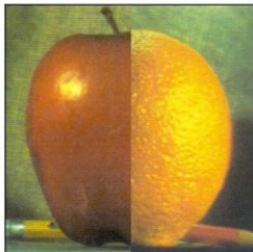
Apple



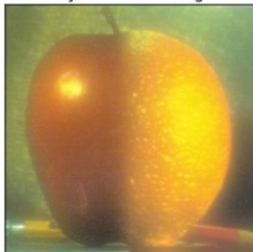
Orange



Direct Connection



Pyramid Blending



HW3

Algorithm for image compositing

- Generate Laplacian pyramid L_o of the orange image

HW3

Algorithm for image compositing

- Generate Laplacian pyramid L_o of the orange image
- Generate Laplacian pyramid L_a of the apple image

HW3

Algorithm for image compositing

- Generate Laplacian pyramid L_o of the orange image
- Generate Laplacian pyramid L_a of the apple image
- Generate Laplacian pyramid L_c by
 - Copy left half of nodes at each level from apple pyramid
 - Copy right half of nodes at each level from orange pyramid

HW3

Algorithm for image compositing

- Generate Laplacian pyramid L_o of the orange image
- Generate Laplacian pyramid L_a of the apple image
- Generate Laplacian pyramid L_c by
 - Copy left half of nodes at each level from apple pyramid
 - Copy right half of nodes at each level from orange pyramid
- Reconstruct combined image from pyramid L_c