ECE 4973/5973: Lecture 5
Spatial Frequencies in Images

Samuel Cheng

Slide credits: James Thompkin

Fourier series

Jean Baptiste Joseph Fourier (1768-1830)

A bold idea (1807):

Any *univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

Hays

Fourier series

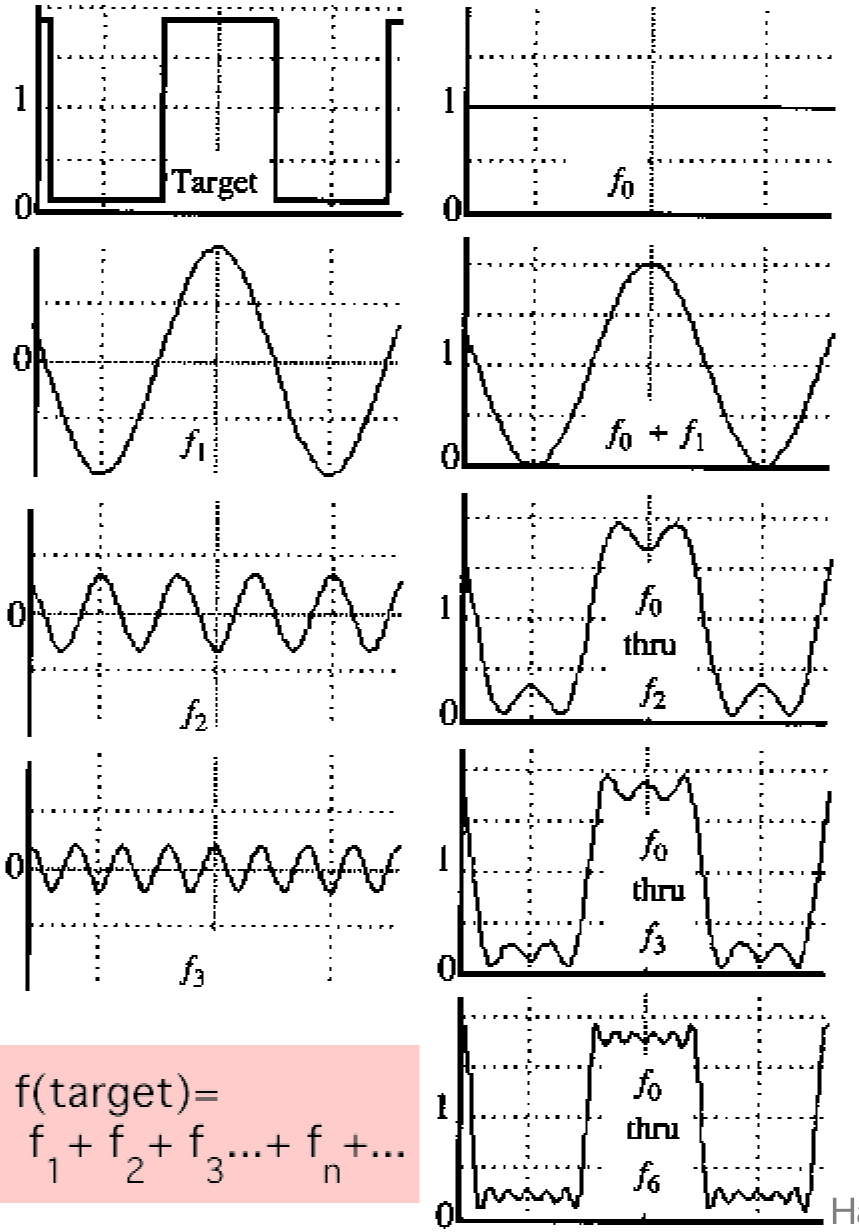
A bold idea (1807):

***Any** univariate function can be rewritten as a weighted sum of sines and cosines of different frequencies.*

Our building block:

$$A\sin(\omega t) + B\cos(\omega t)$$

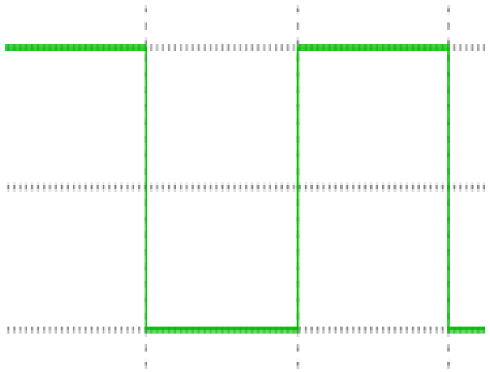
Add enough of them to get any signal *g(t)* you want!



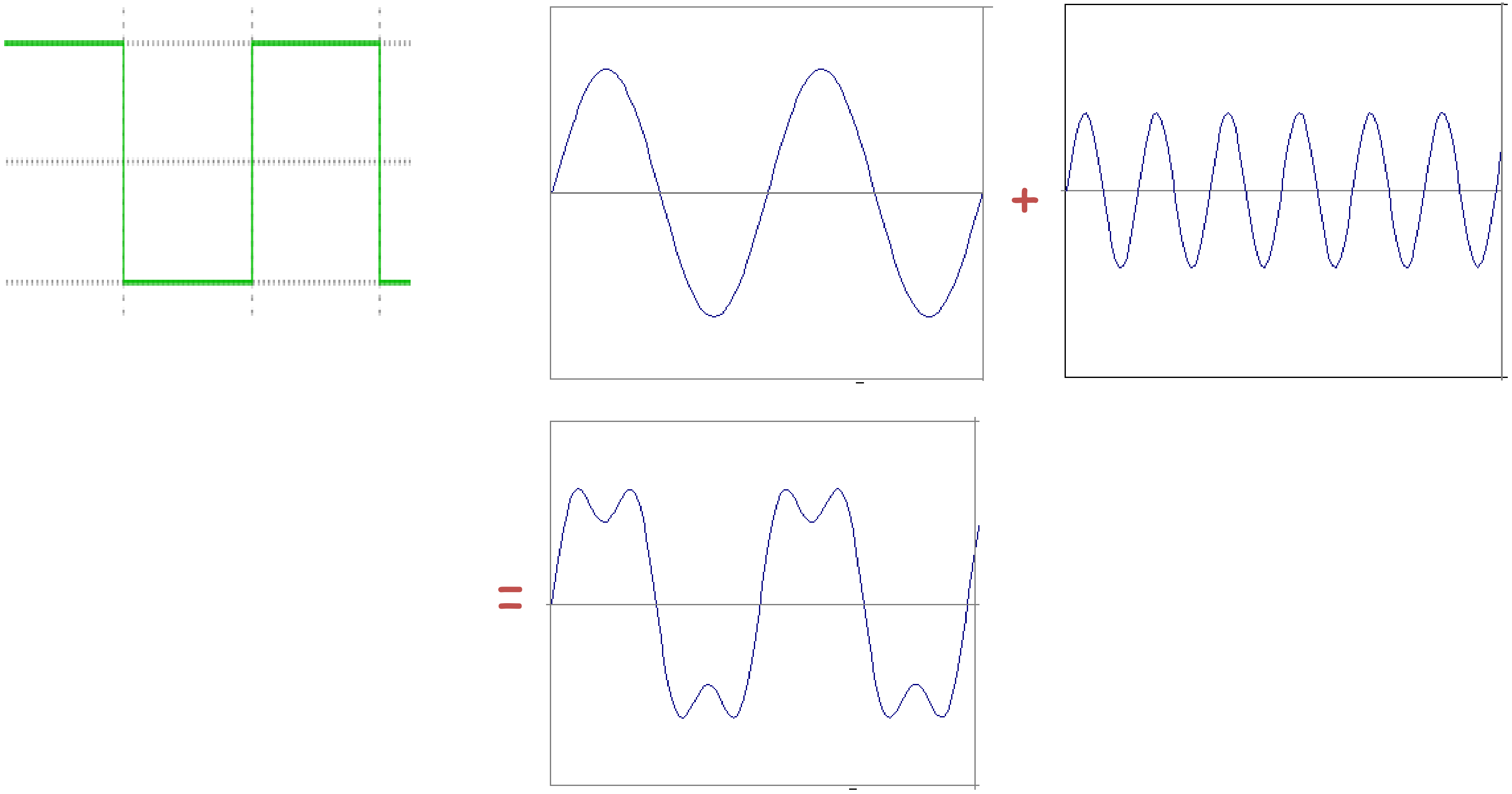
f(target)= $f_1 + f_2 + f_3 ... + f_n + ...$

Hays

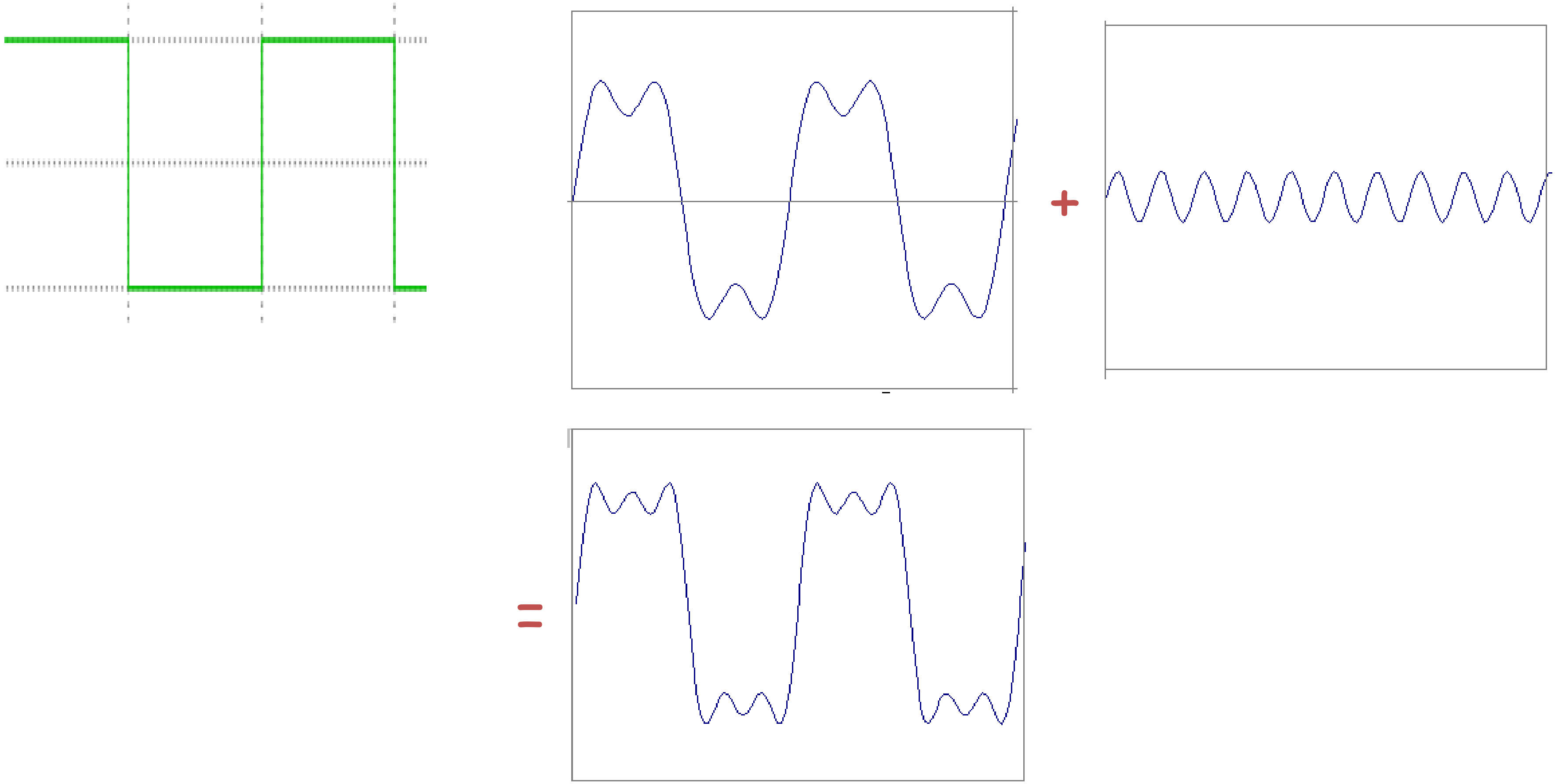
Square wave spectra



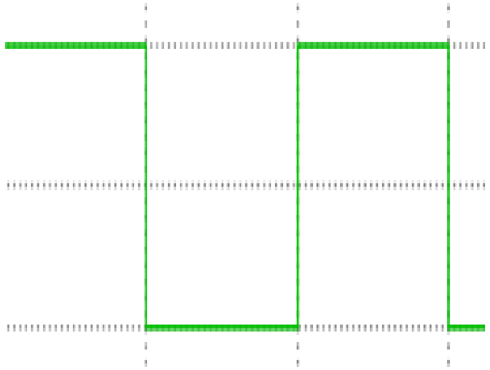
Square wave spectra

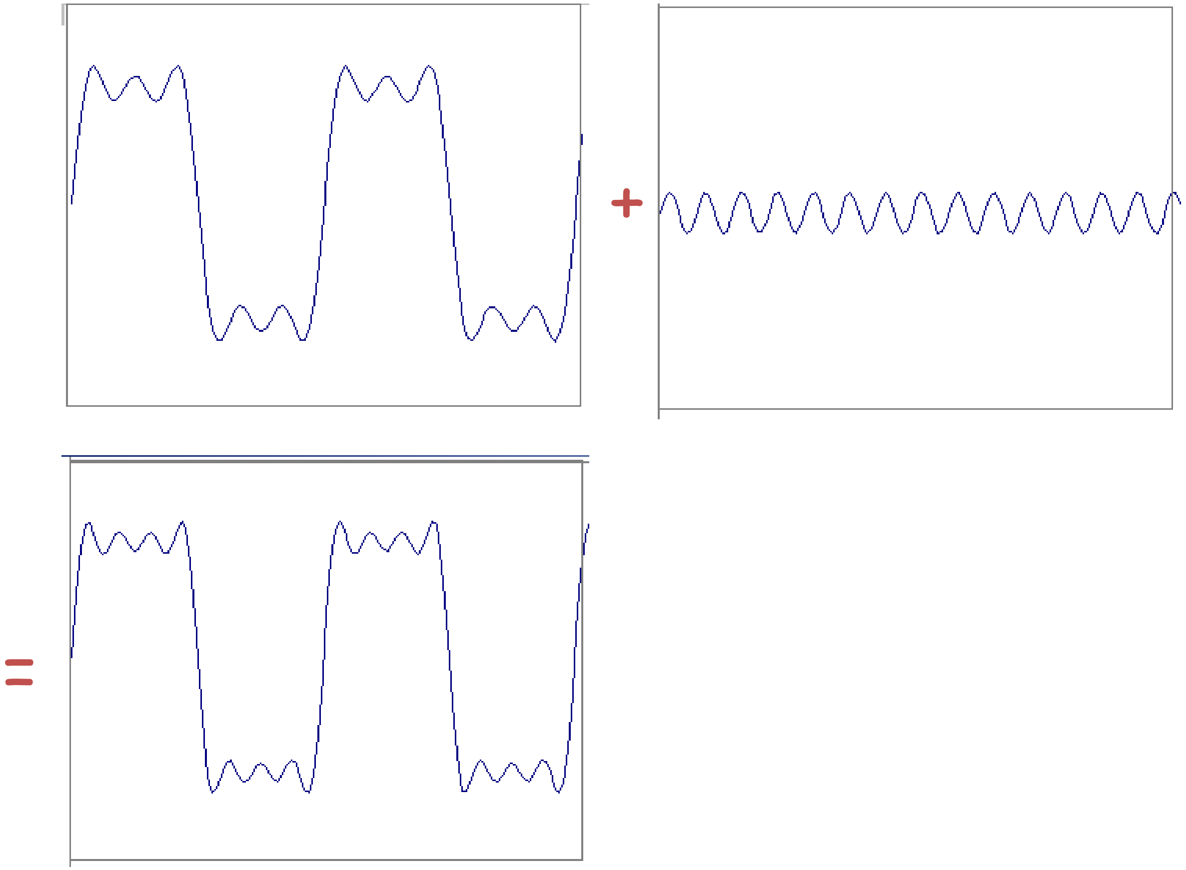


Square wave spectra

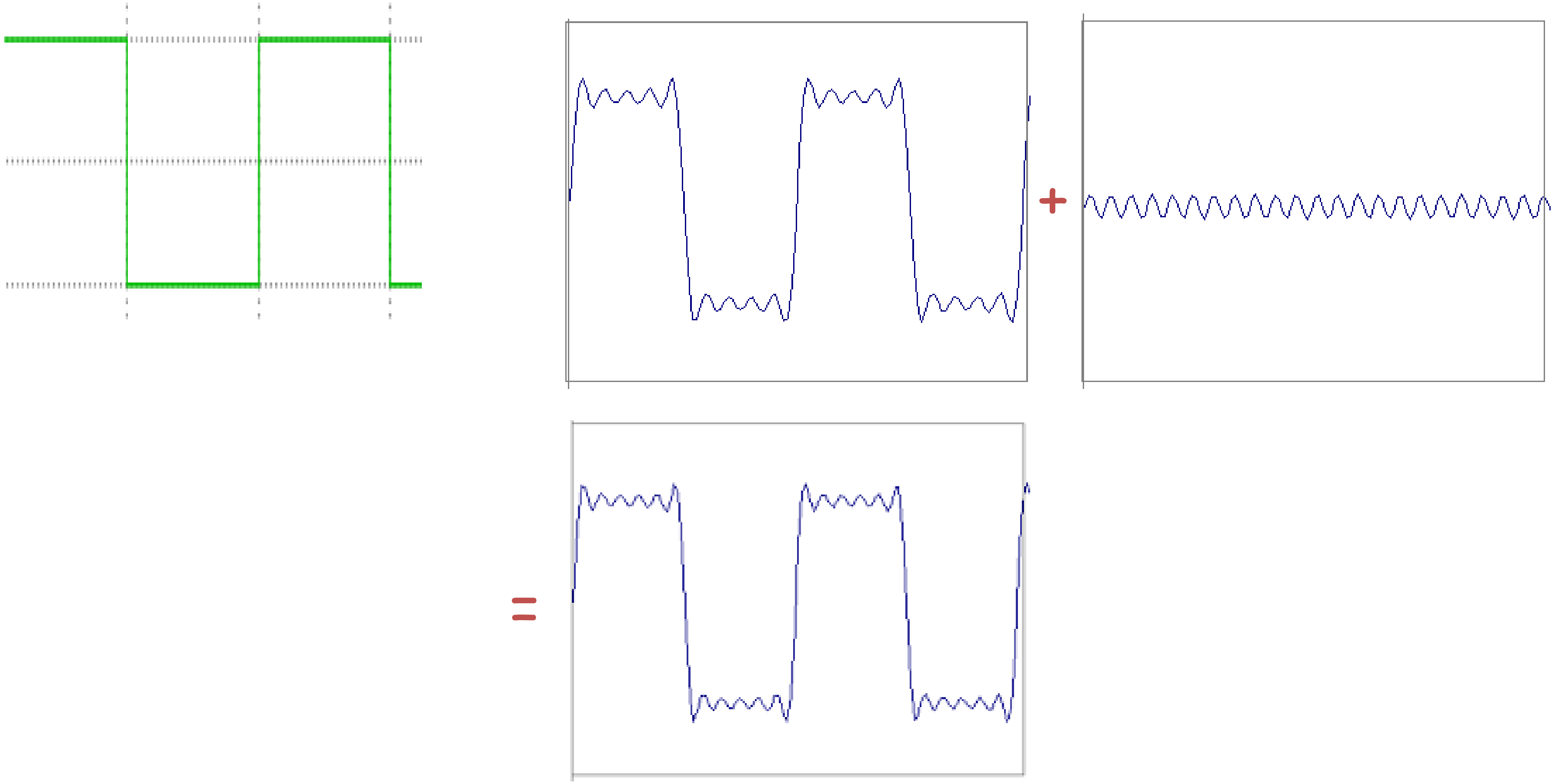


Square wave spectra

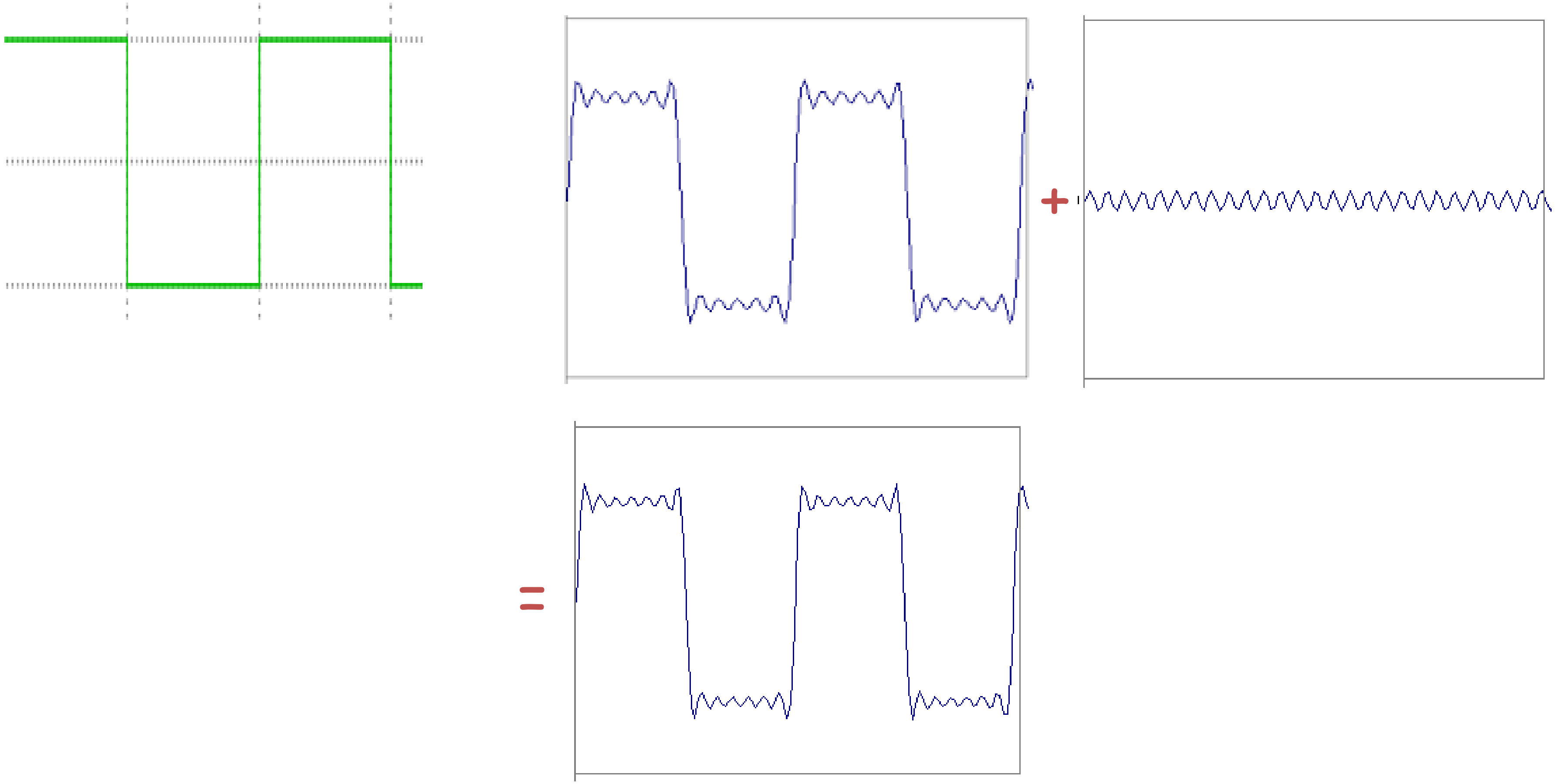




Square wave spectra

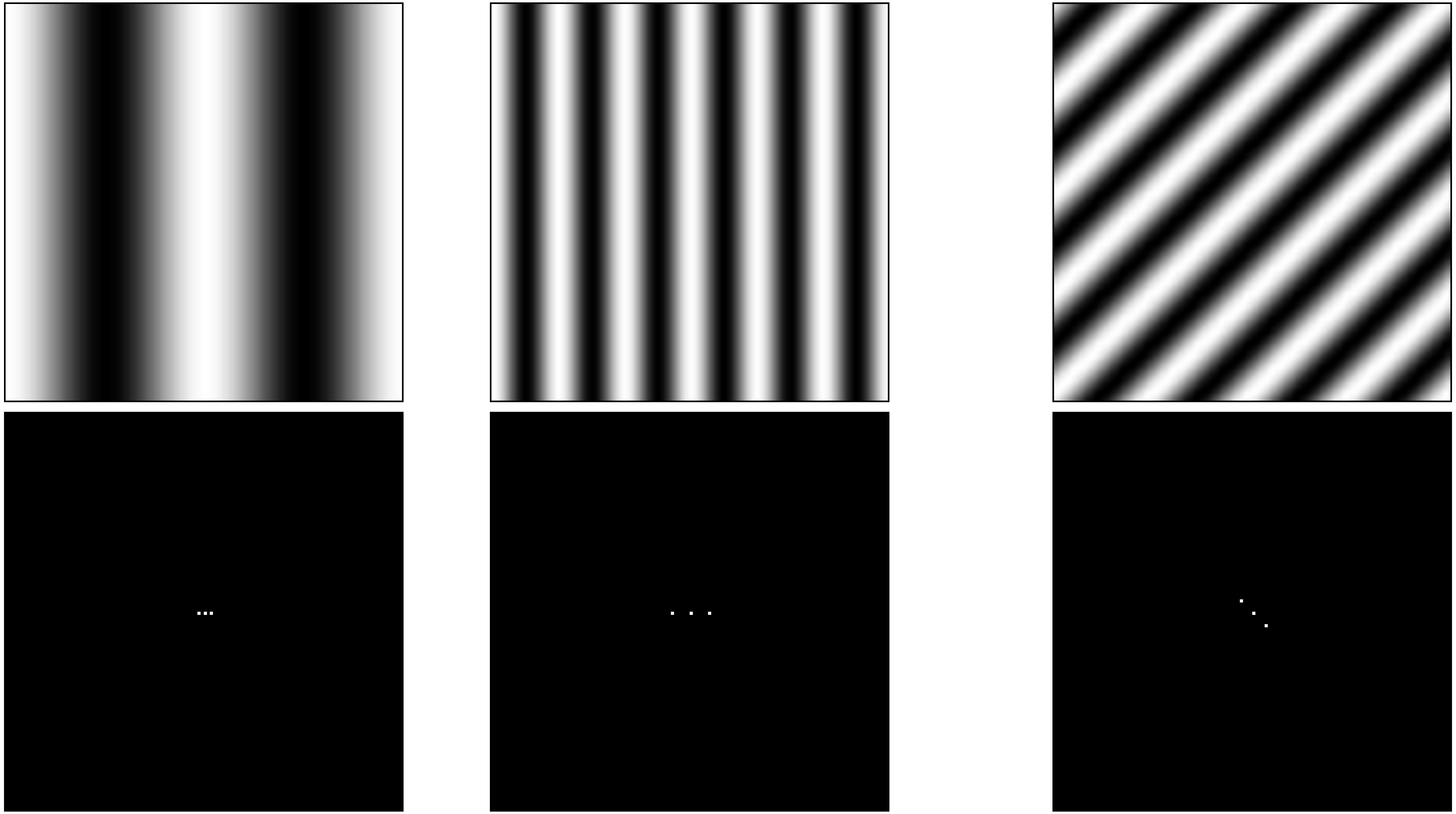


Square wave spectra



Fourier analysis in images

Spatial domain images

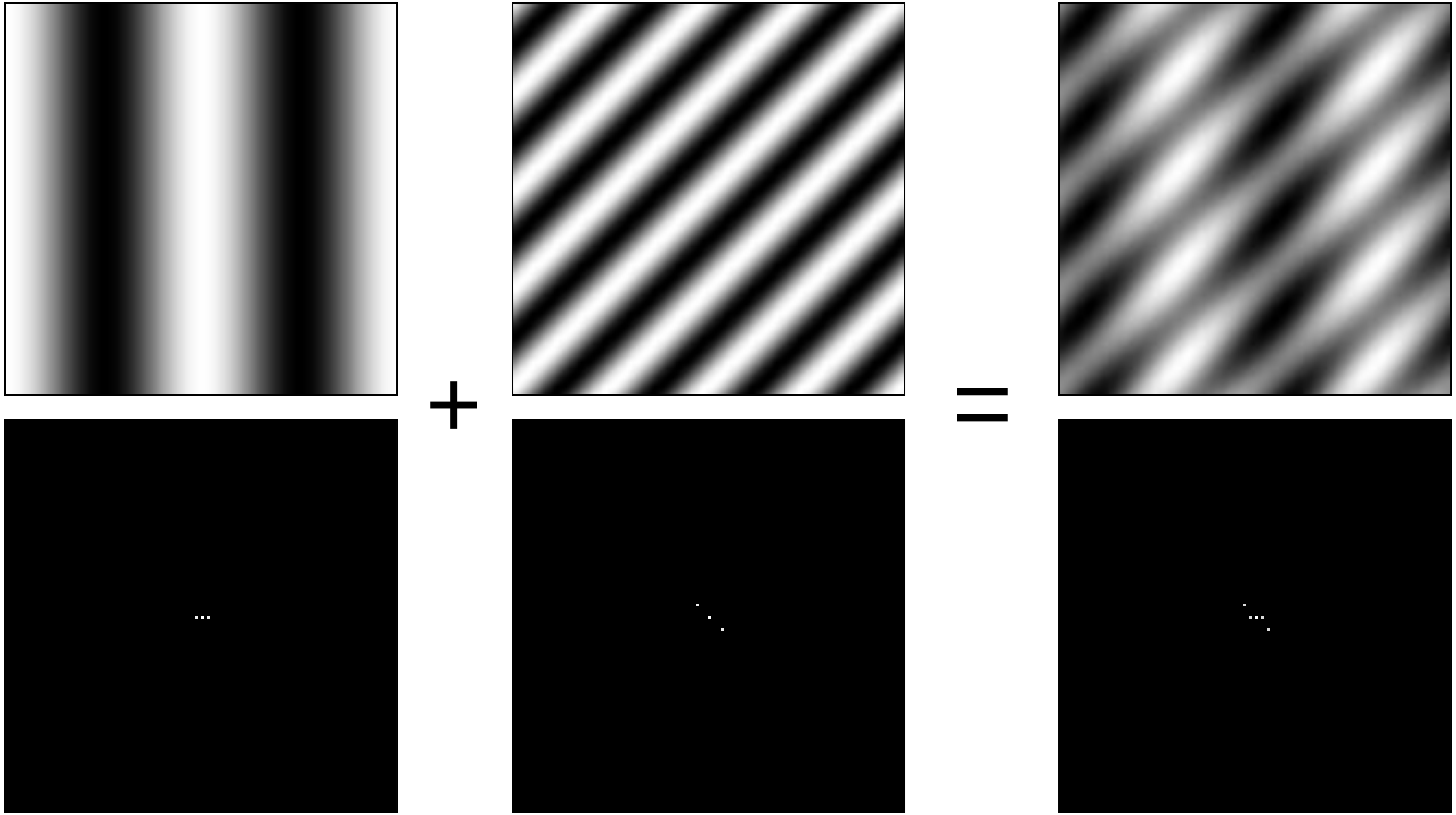


Fourier decomposition frequency amplitude images

http://sharp.bu.edu/~slehar/fourier/fourier.html#filtering More: http://www.cs.unm.edu/~brayer/vision/fourier.html

Signals can be composed

Spatial domain images



Fourier decomposition frequency amplitude images

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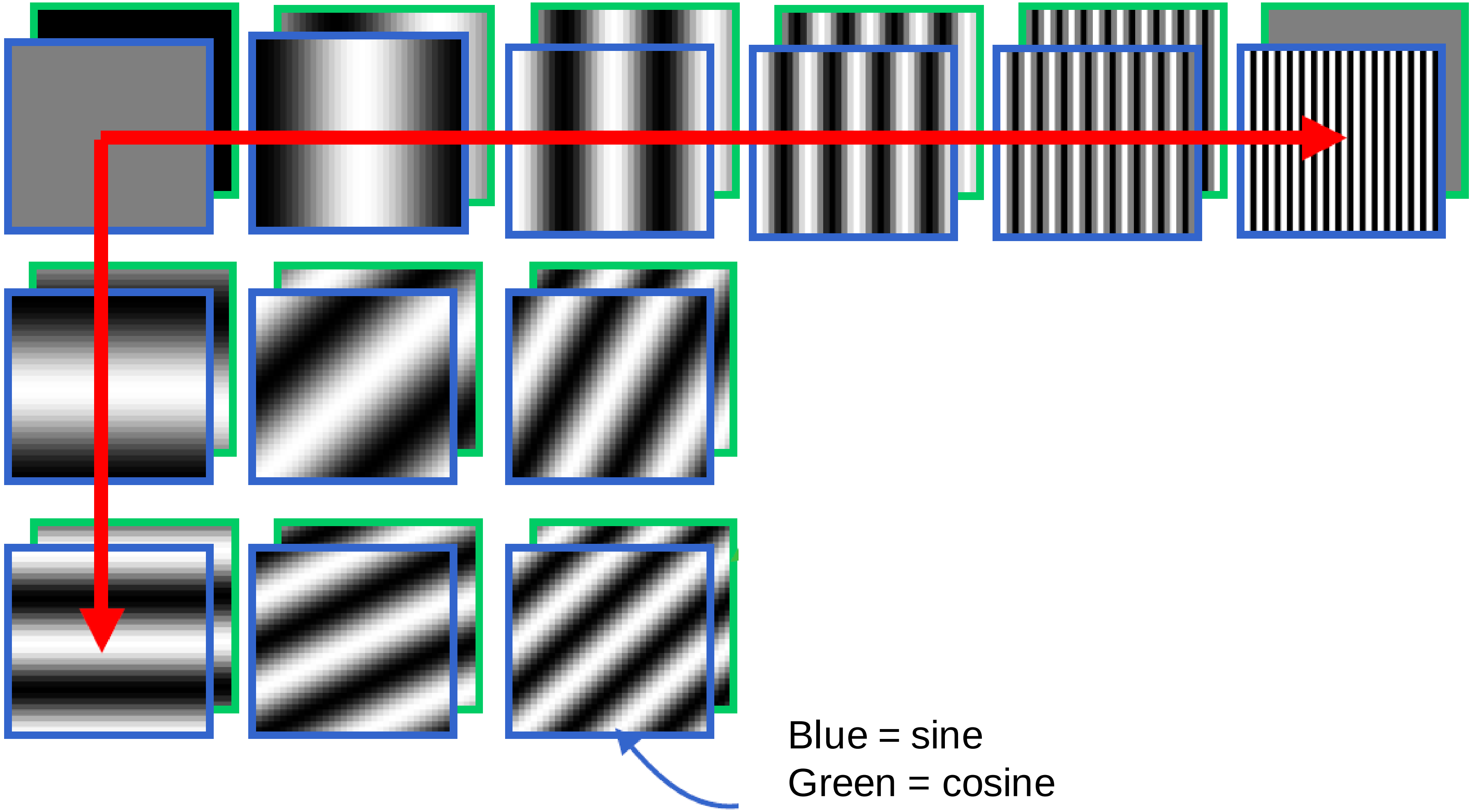
Fourier Bases

Teases away 'fast vs. slow' changes in the image.



Blue = sine
Green = cosine

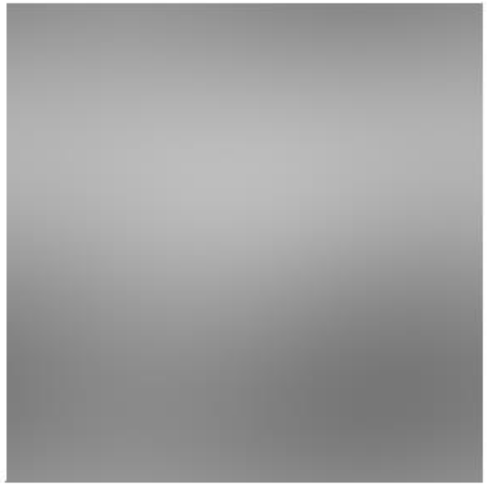
This change of basis is the Fourier Transform

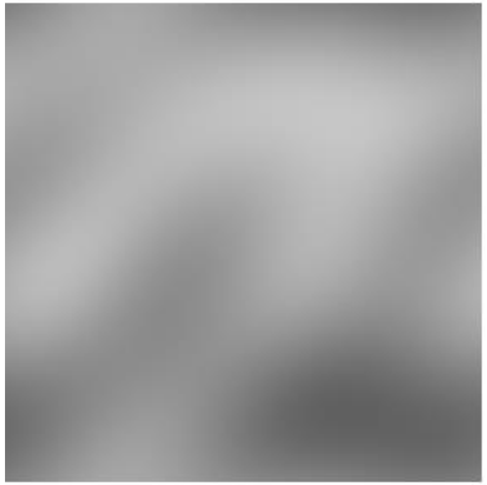
Hays

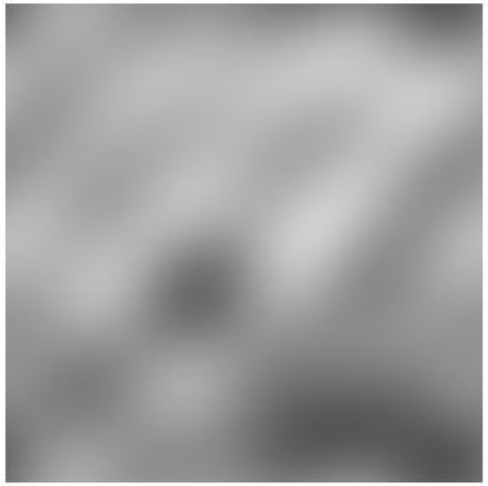
Basis reconstruction

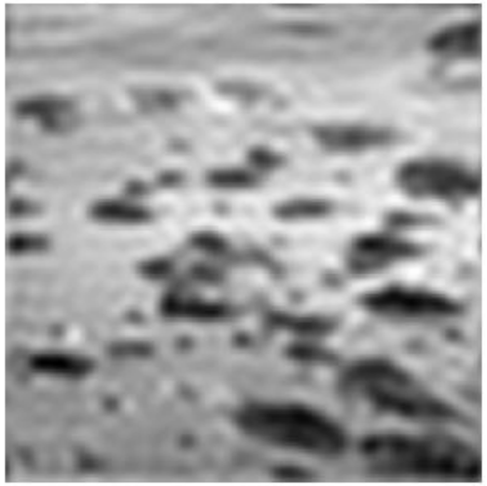

Full image


First 1 basis fn


First 4 basis fns


First 9 basis fns

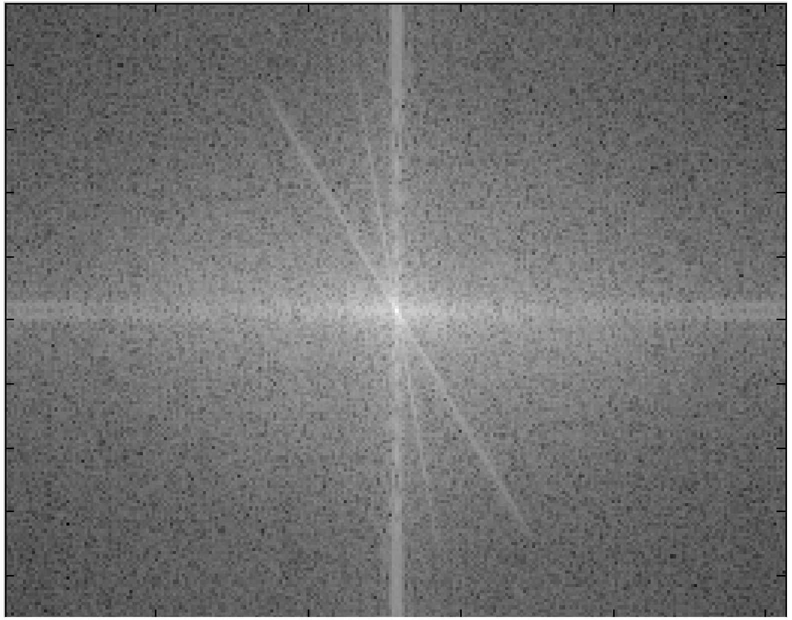

First 16 basis fns


First 400 basis fns

Natural image

Natural image

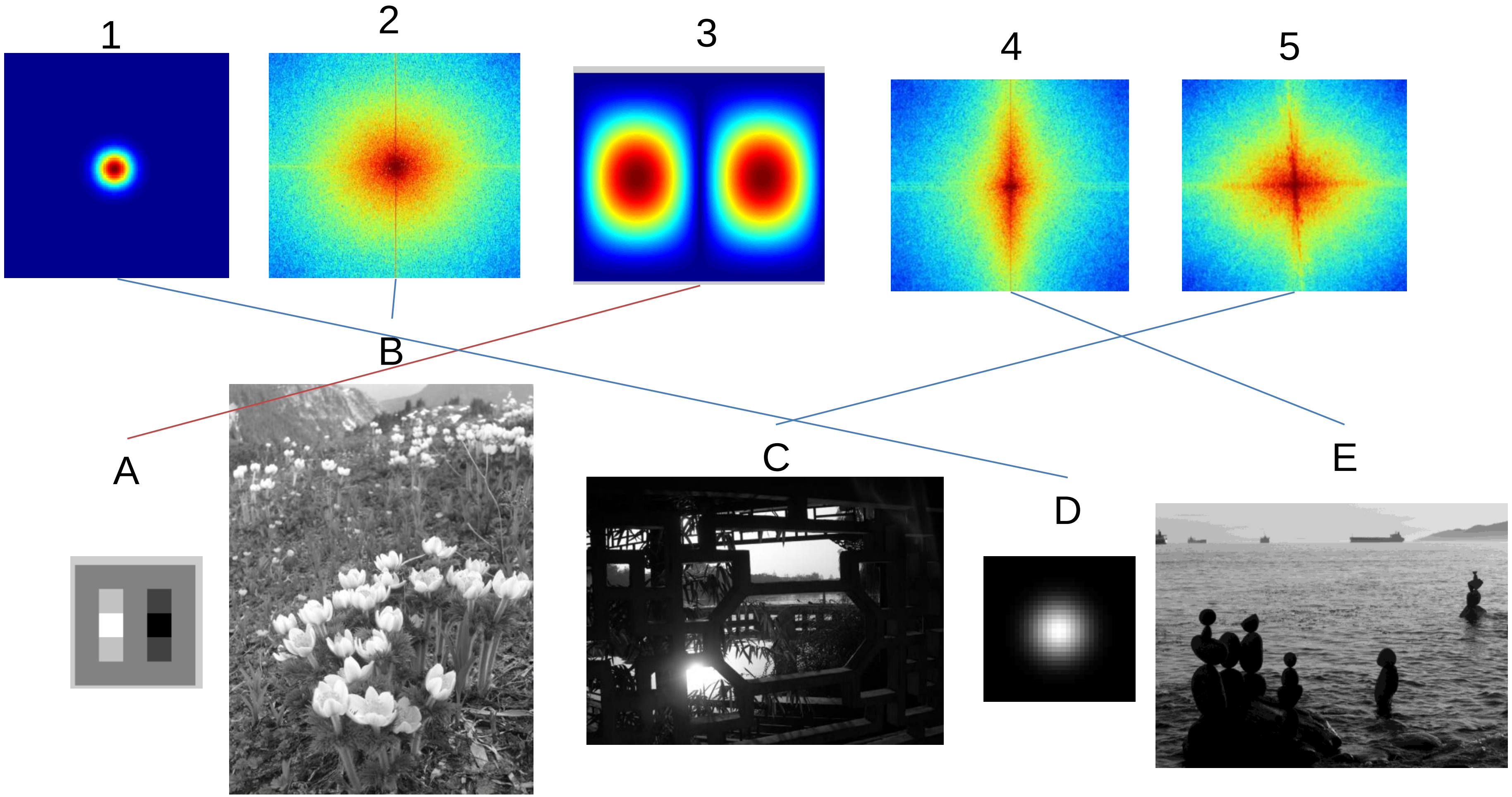
Fourier decomposition
Frequency coefficients (amplitude)



What does it mean to be at pixel x,y?
What does it mean to be more or less bright in the Fourier decomposition image?

Think-Pair-Share

Match the spatial domain image to the Fourier magnitude image



Hoiem

Fourier Transform

- Stores the amplitude and phase at each frequency:
  - For mathematical convenience, this is often notated in terms of real and complex numbers
  - Related by Euler's formula

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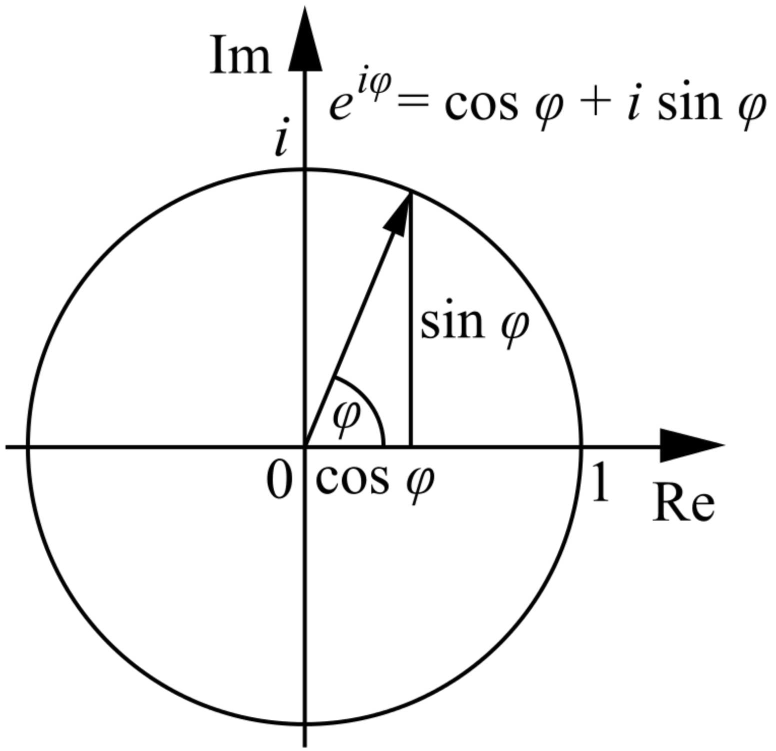

Fourier Transform

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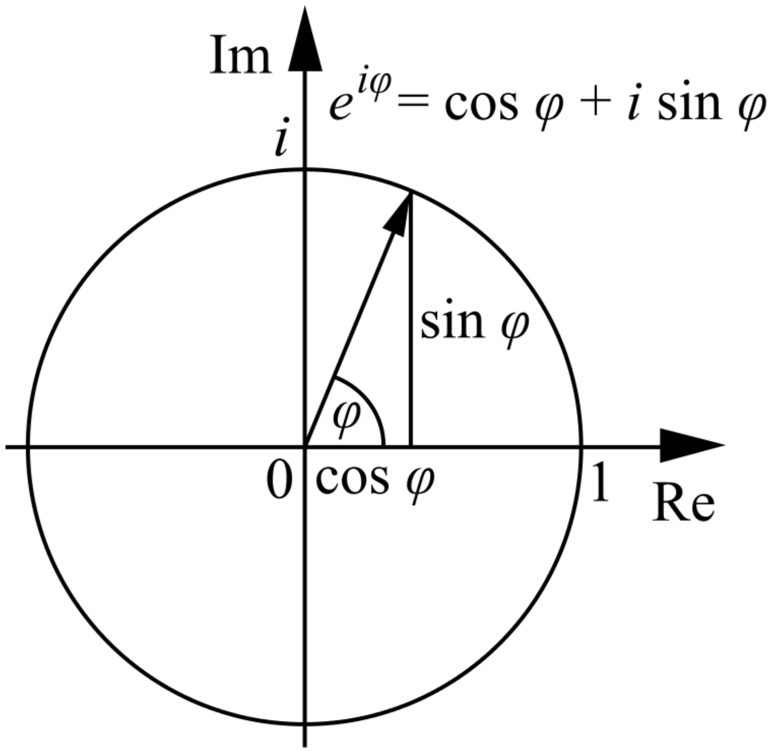


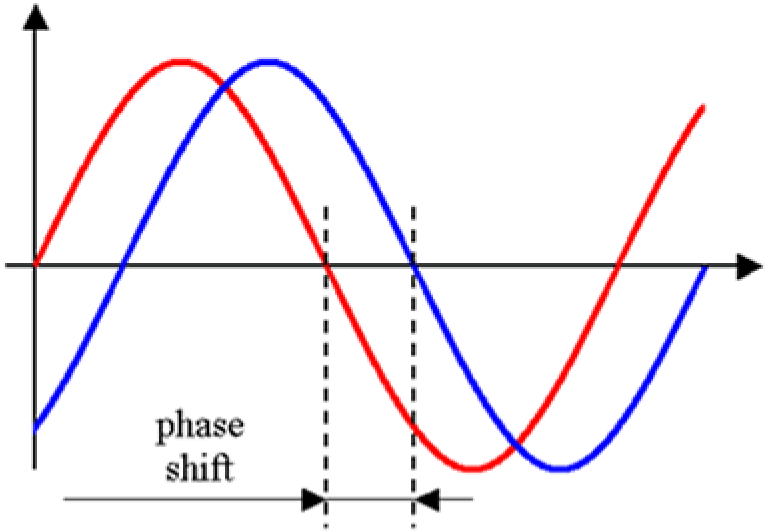
Amplitude encodes how much signal there is at a particular frequency:

$$A = \pm\sqrt{\mathrm{Re}(\varphi)^2 + \mathrm{Im}(\varphi)^2}$$

Phase encodes spatial information (indirectly):

$$\phi = \tan^{-1}\frac{\mathrm{Im}(\varphi)}{\mathrm{Re}(\varphi)}$$

Amplitude / Phase



- Amplitude tells you "how much"
- Phase tells you "where"

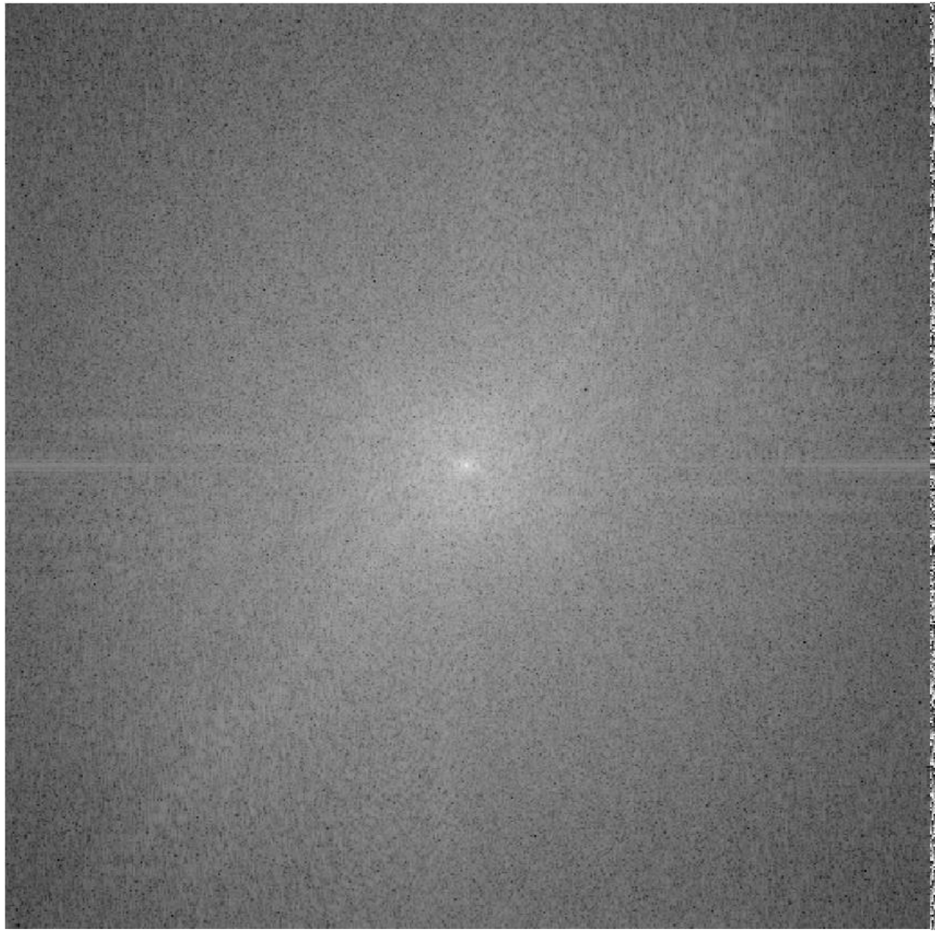
- Translate the image?
  - Amplitude unchanged
  - Adds a constant to the phase.

What about phase?

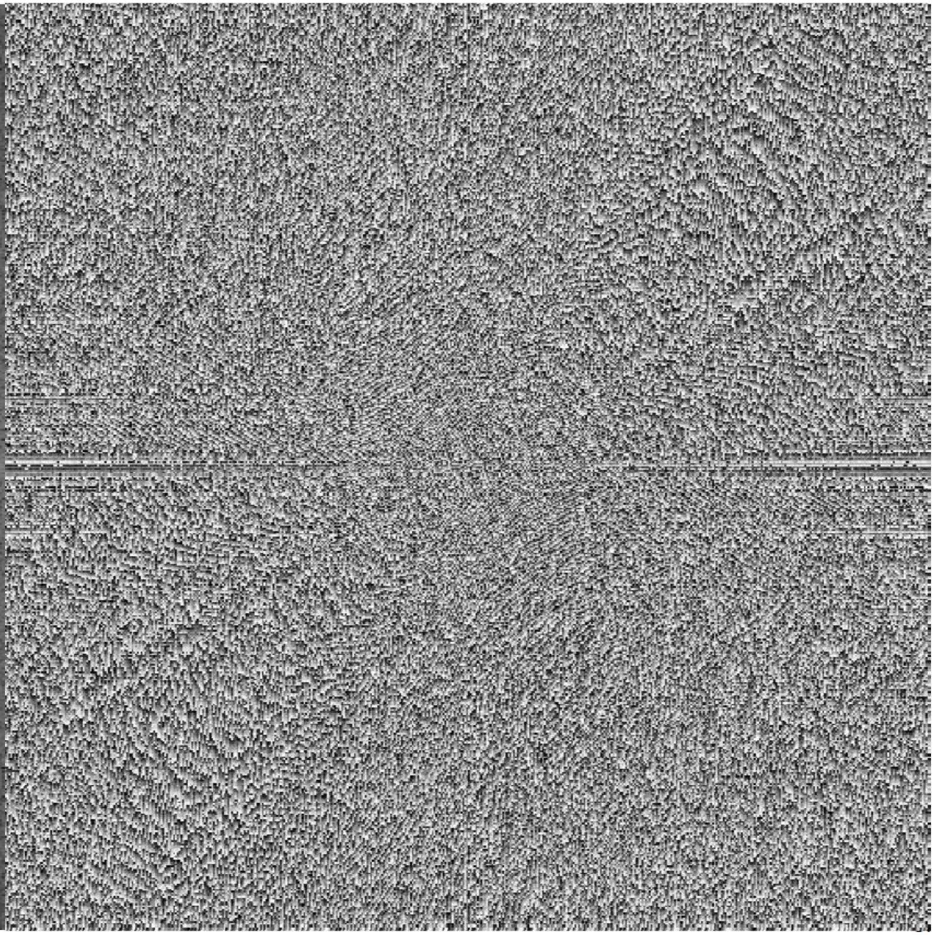
Efros

What about phase?

Amplitude



Phase



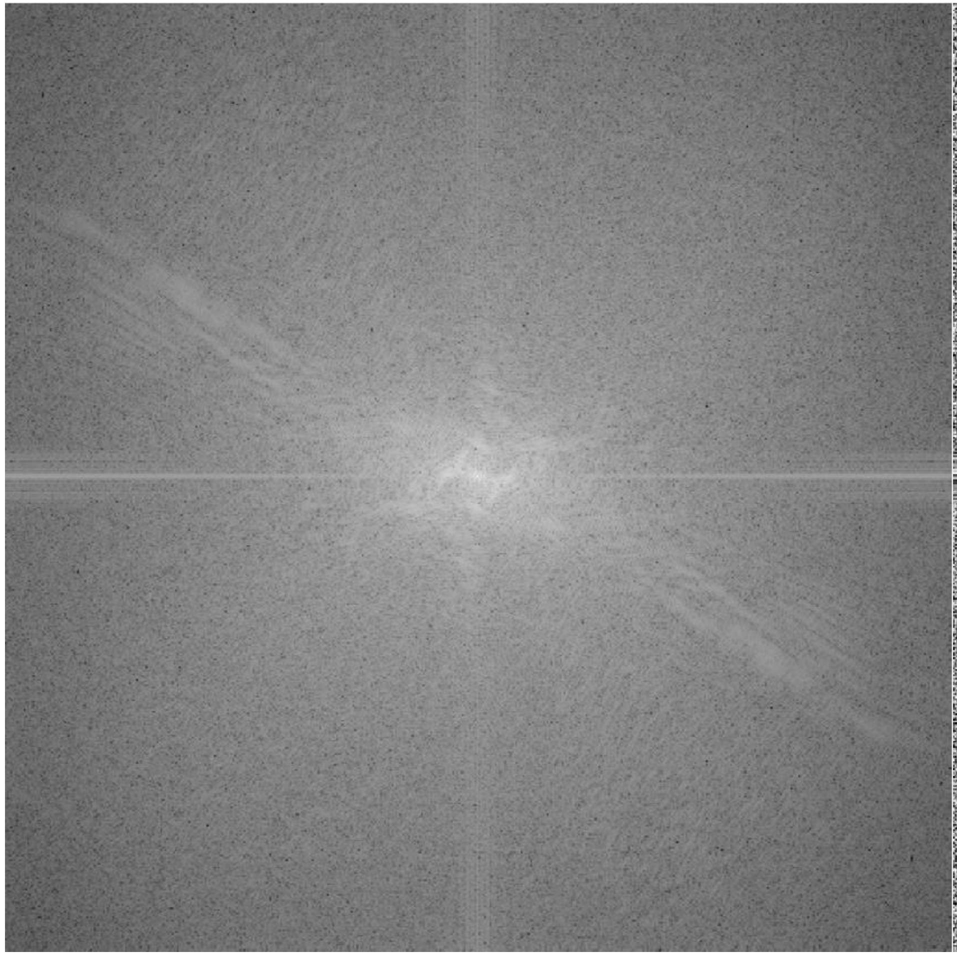
Efros

What about phase?

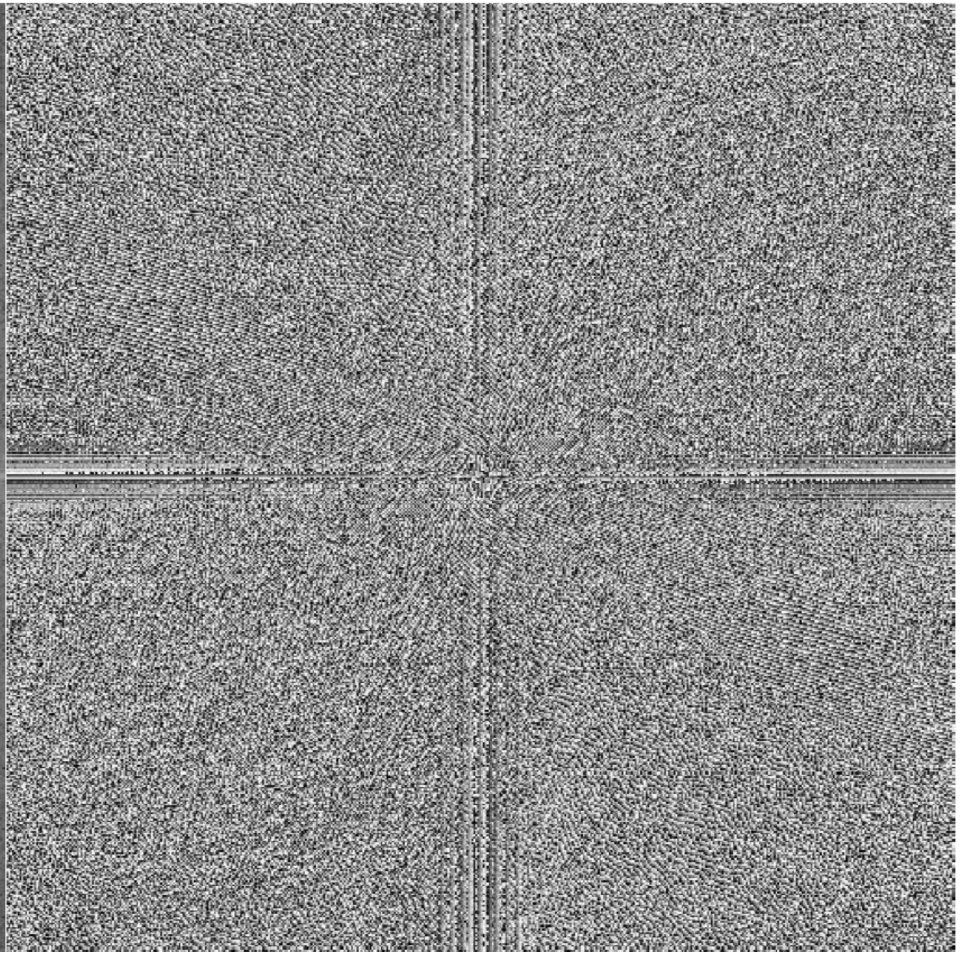
Efros

What about phase?

Amplitude



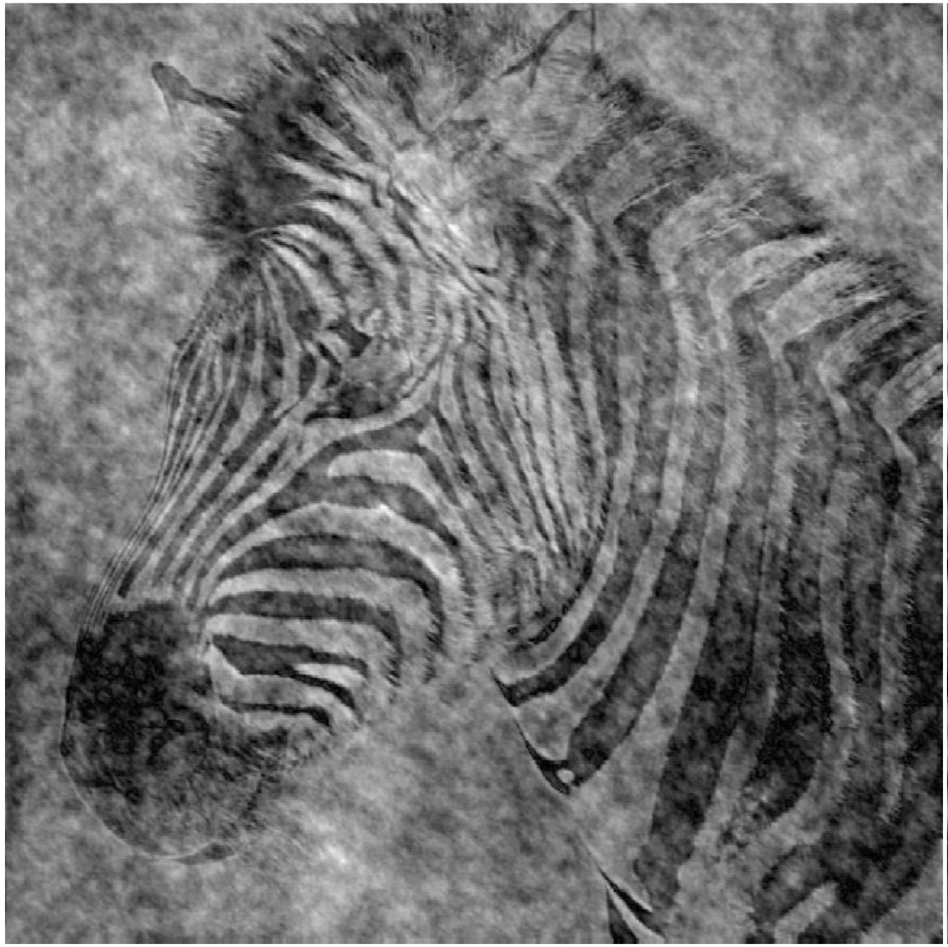
Phase



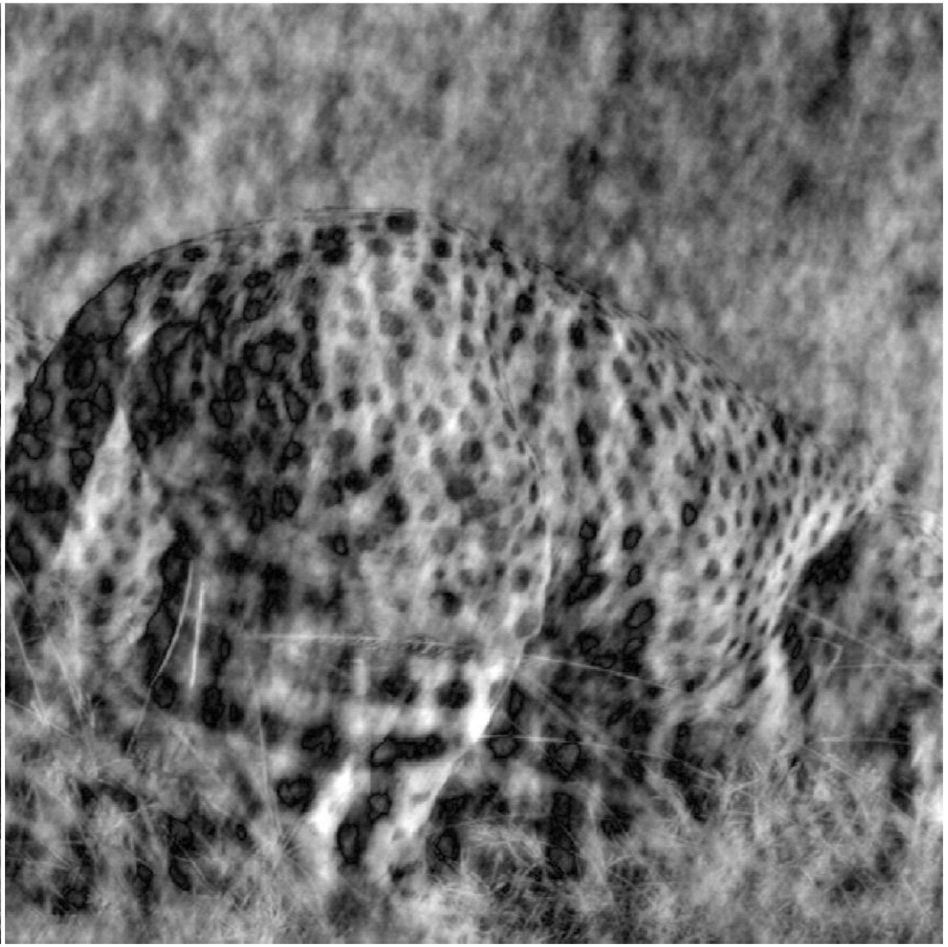
Efros

Cheebra

Zebra phase, cheetah amplitude



Cheetah phase, zebra amplitude



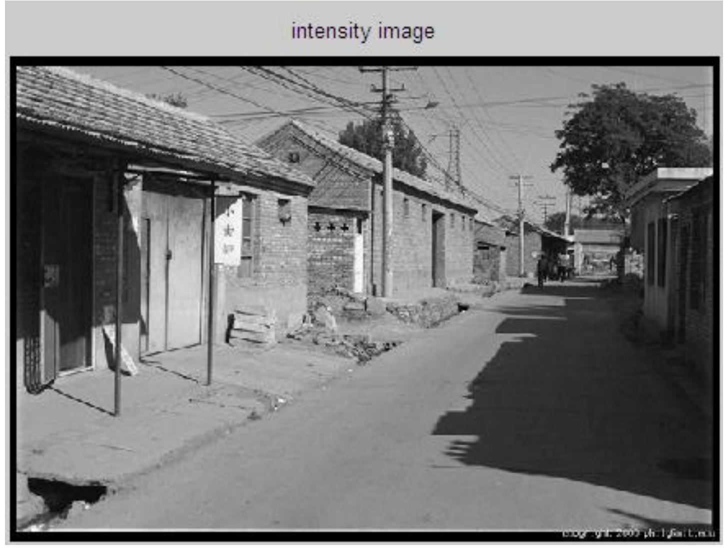
Efros

- The frequency amplitude of natural images are quite similar
  - Heavy in low frequencies, falling off in high frequencies
- Most information in the image is “carried” in the phase, not the amplitude
  - Not quite clear why

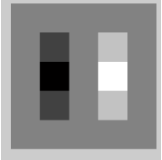
Efros

Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1



*



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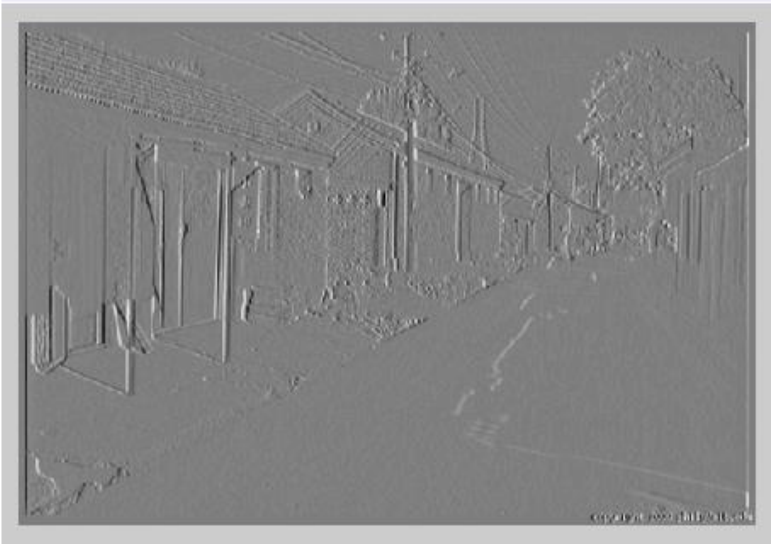
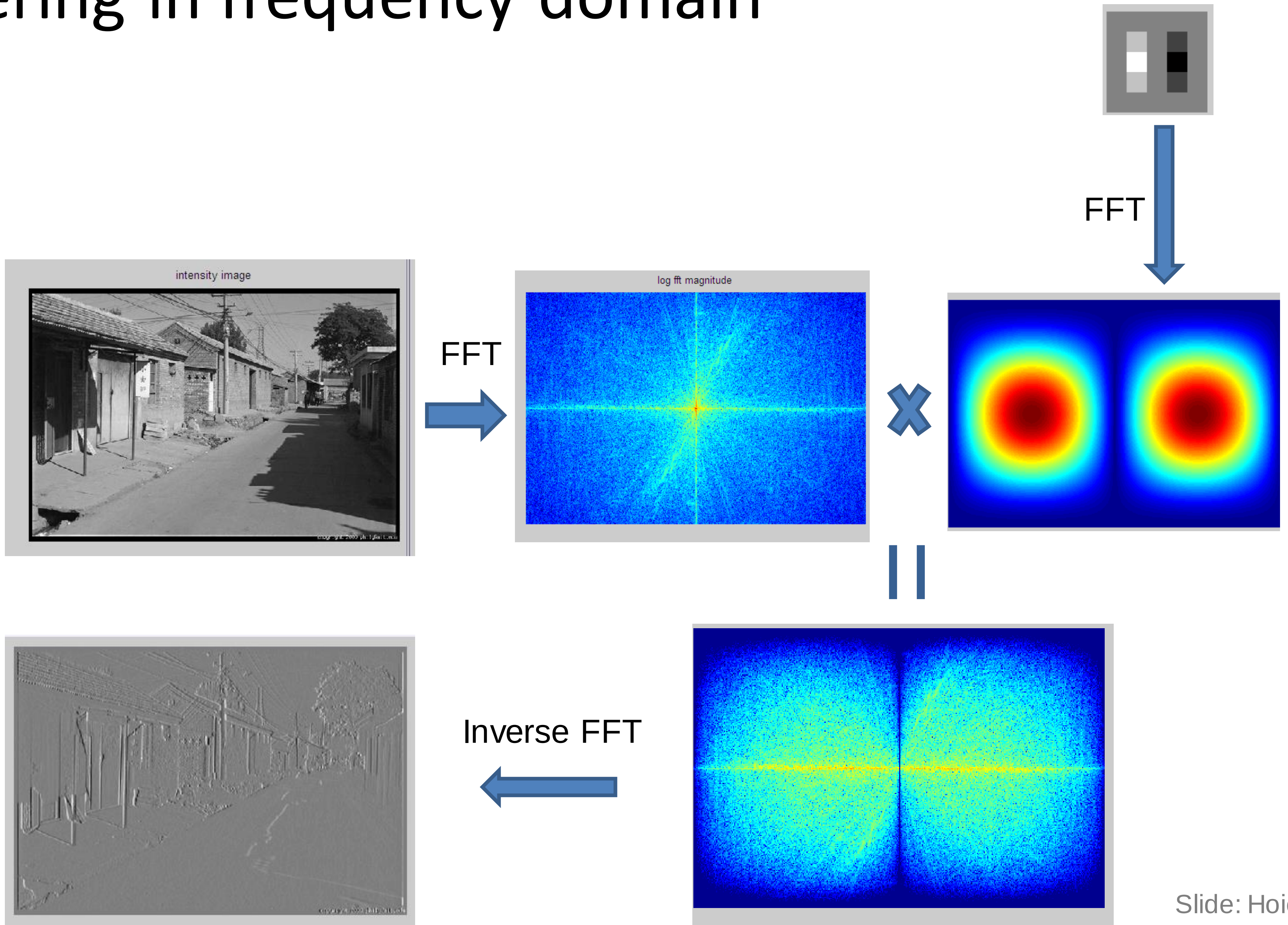


Hays

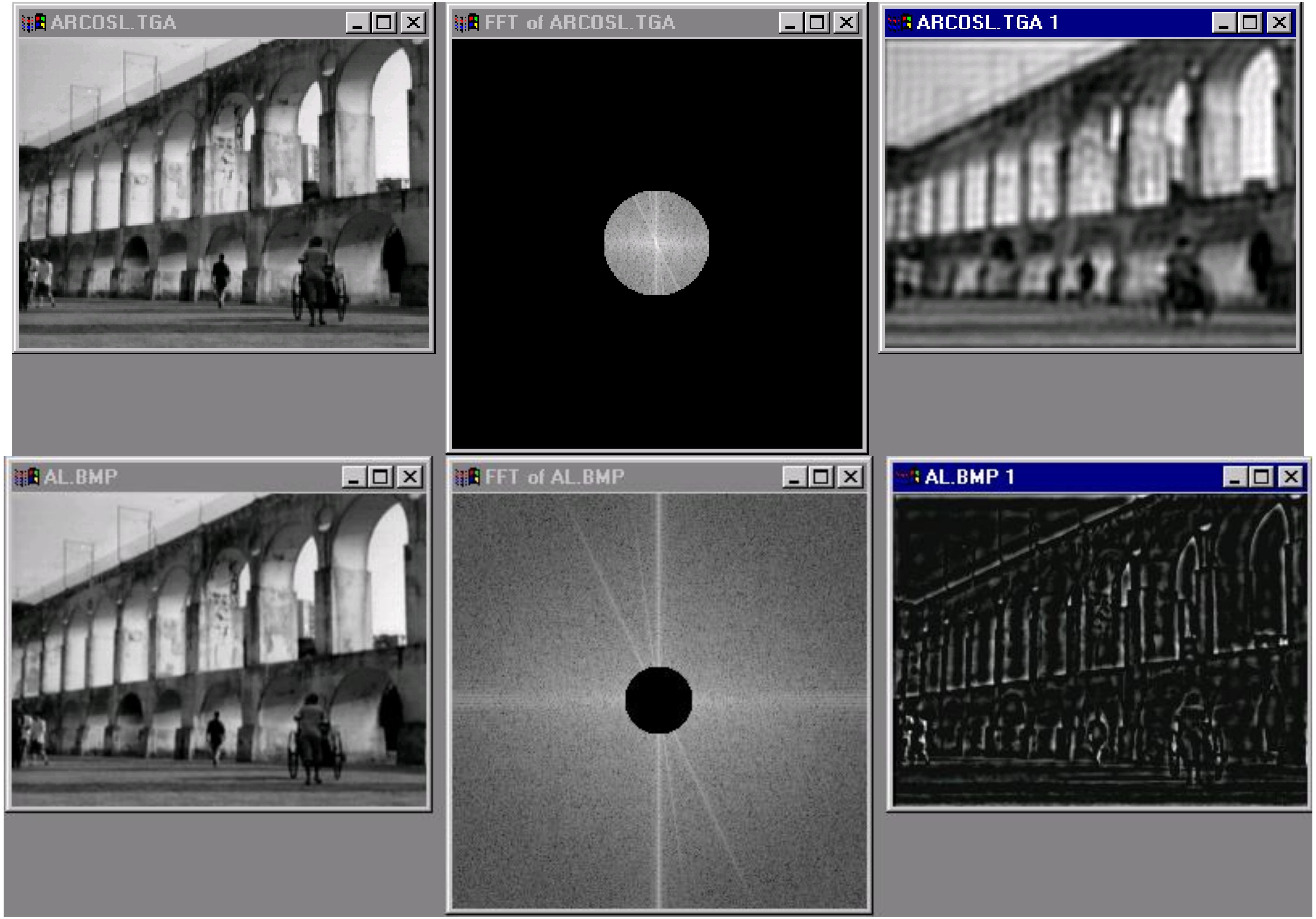
Filtering in frequency domain



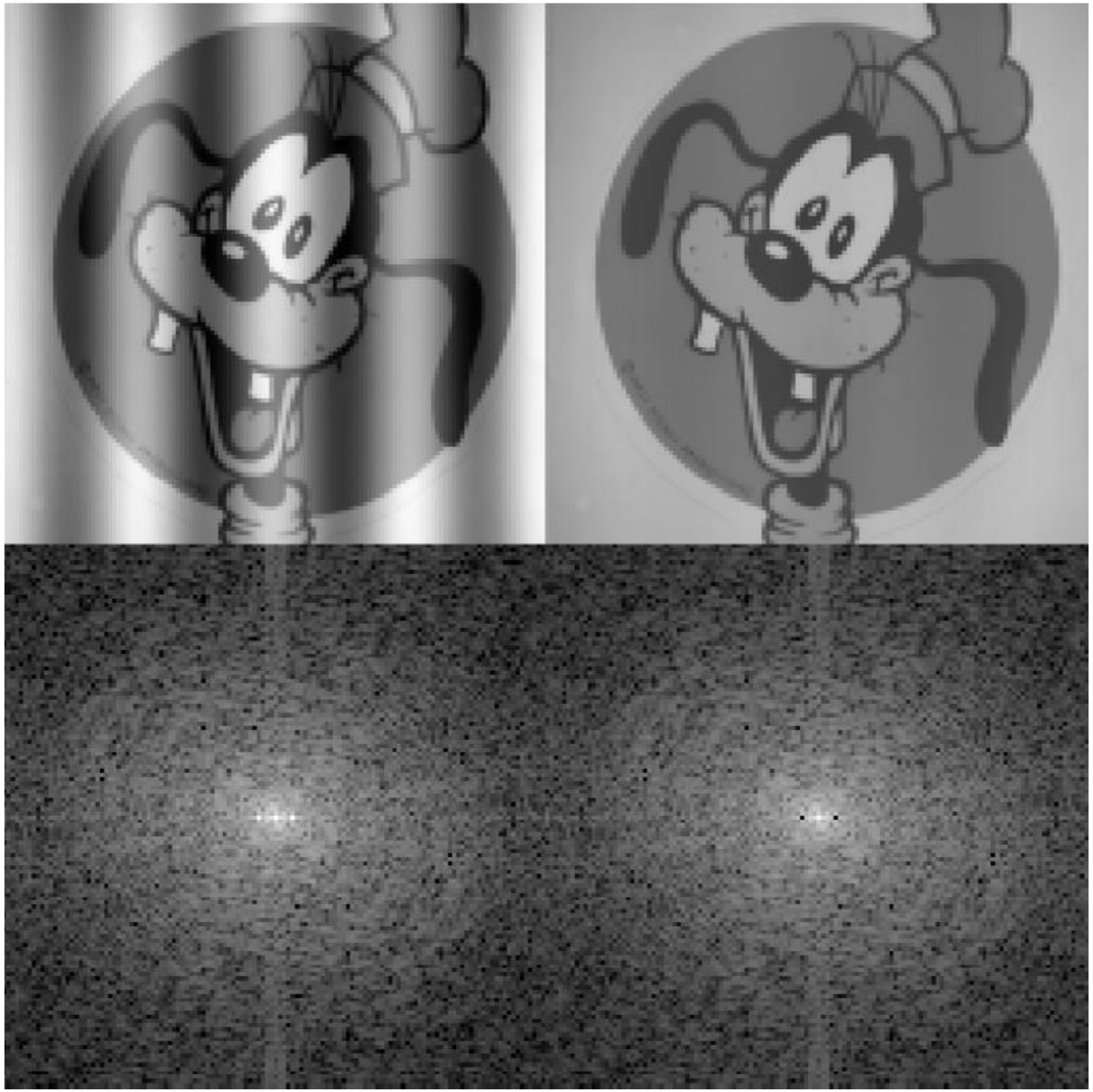
Slide: Hoiem

Now we can edit frequencies!

Low and High Pass filtering



Removing frequency bands

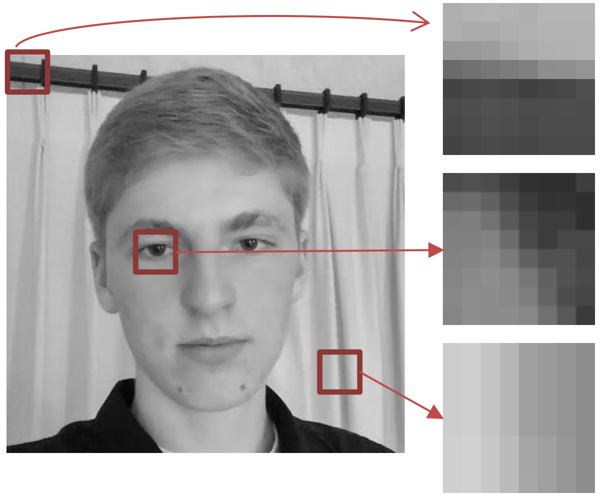


Brayer

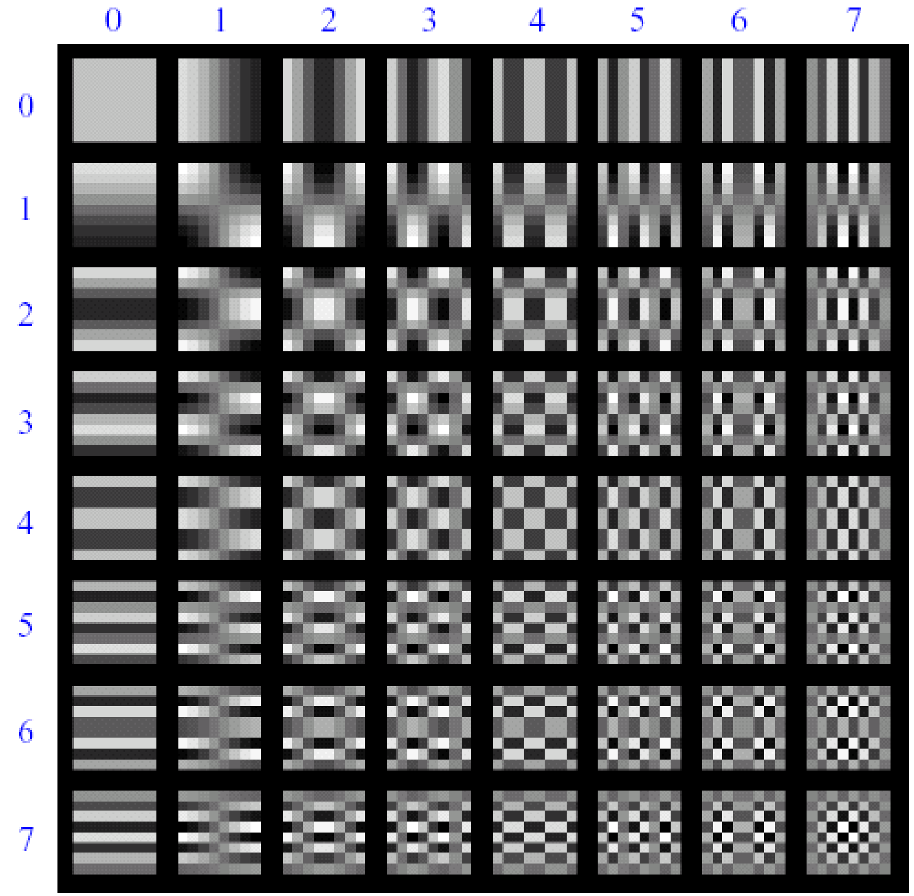
JPEG Image Compression

Lossy Image Compression (JPEG)

8x8 blocks



8x8 blocks



Block-based Discrete Cosine Transform (DCT)

Slides: Efros

Image compression using DCT

- Compute DCT filter responses in each 8x8 block

Filter responses

$$G = \begin{bmatrix} -415.38 & -30.19 & -61.20 & 27.24 & 56.13 & -20.10 & -2.39 & 0.46 \\ 4.47 & -21.86 & -60.76 & 10.25 & 13.15 & -7.09 & -8.54 & 4.88 \\ -46.83 & 7.37 & 77.13 & -24.56 & -28.91 & 9.93 & 5.42 & -5.65 \\ -48.53 & 12.07 & 34.10 & -14.76 & -10.24 & 6.30 & 1.83 & 1.95 \\ 12.12 & -6.55 & -13.20 & -3.95 & -1.88 & 1.75 & -2.79 & 3.14 \\ -7.73 & 2.91 & 2.38 & -5.94 & -2.38 & 0.94 & 4.30 & 1.85 \\ -1.03 & 0.18 & 0.42 & -2.42 & -0.88 & -3.02 & 4.12 & -0.66 \\ -0.17 & 0.14 & -1.07 & -4.19 & -1.17 & -0.10 & 0.50 & 1.68 \end{bmatrix} \quad \overset{u}{\longrightarrow}, \; \downarrow v$$

Quantization divisers (element-wise)

$$Q = \begin{bmatrix} 16 & 11 & 10 & 16 & 24 & 40 & 51 & 61 \\ 12 & 12 & 14 & 19 & 26 & 58 & 60 & 55 \\ 14 & 13 & 16 & 24 & 40 & 57 & 69 & 56 \\ 14 & 17 & 22 & 29 & 51 & 87 & 80 & 62 \\ 18 & 22 & 37 & 56 & 68 & 109 & 103 & 77 \\ 24 & 35 & 55 & 64 & 81 & 104 & 113 & 92 \\ 49 & 64 & 78 & 87 & 103 & 121 & 120 & 101 \\ 72 & 92 & 95 & 98 & 112 & 100 & 103 & 99 \end{bmatrix}$$

Quantized values

$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

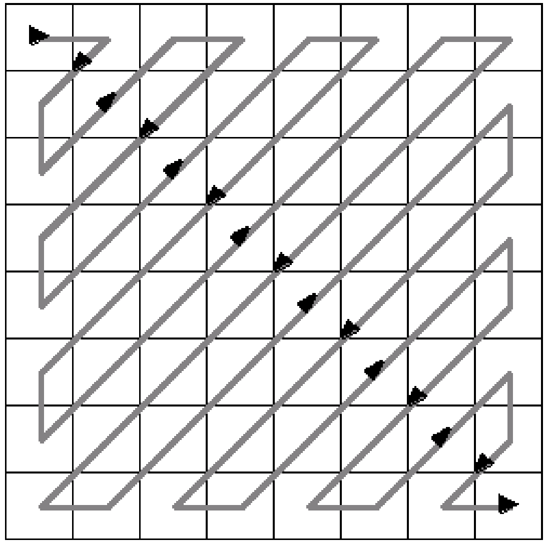
JPEG Encoding

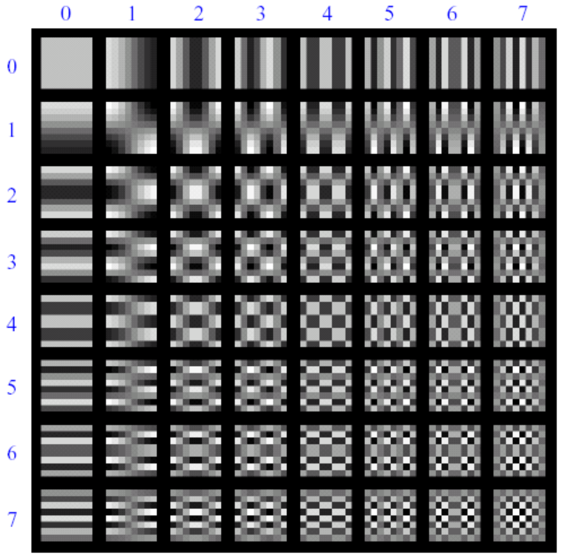
- Entropy coding (Huffman-variant)

Quantized values

$$B = \begin{bmatrix} -26 & -3 & -6 & 2 & 2 & -1 & 0 & 0 \\ 0 & -2 & -4 & 1 & 1 & 0 & 0 & 0 \\ -3 & 1 & 5 & -1 & -1 & 0 & 0 & 0 \\ -3 & 1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

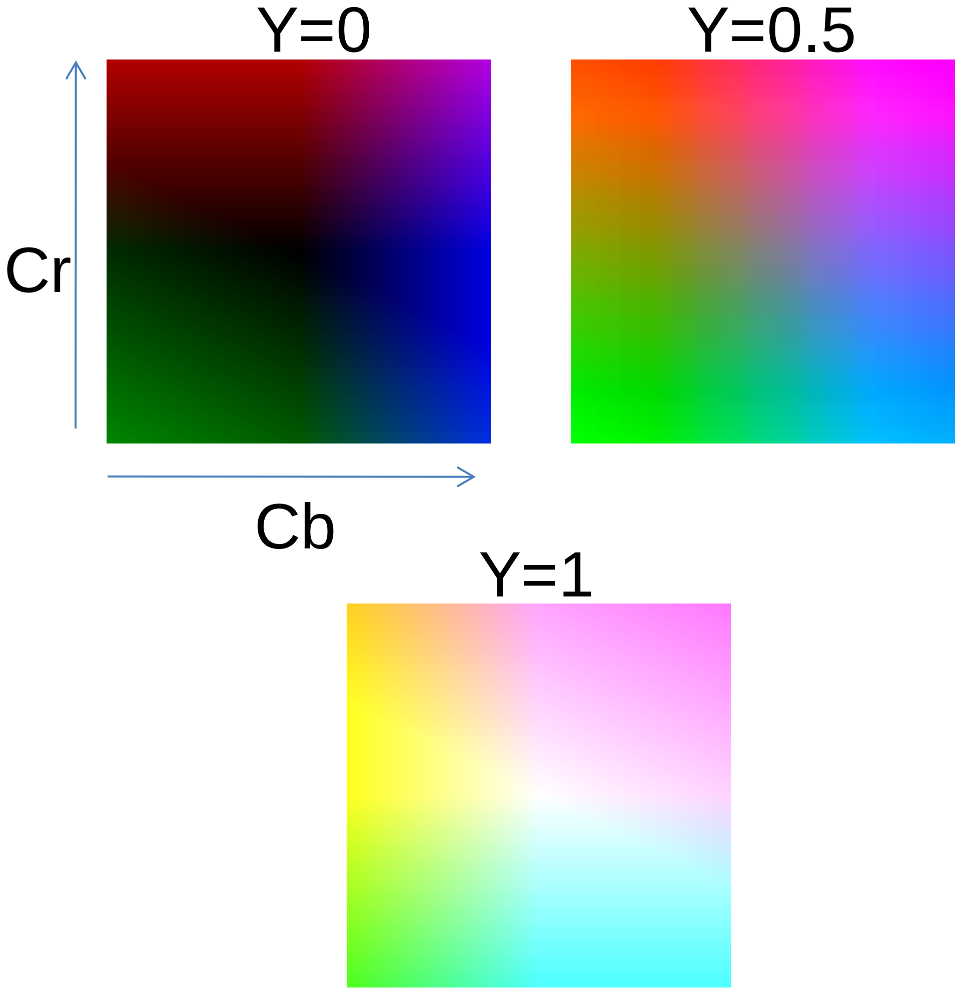
Linearize B like this.





Color spaces: YCbCr

Fast to compute, good for compression, used by TV



Y
(Cb=0.5,Cr=0.5)

Cb
(Y=0.5,Cr=0.5)

Cr
(Y=0.5,Cb=05)

James Hays

Most JPEG images & videos subsample chroma

PSP Comp 3
2x2 Chroma subsampling
285K

Original
1,261K lossless
968K PNG

Summary

- Signals and images can be decomposed into sinusoidal components (Fourier analysis/transform/spectral theorem)
  - High frequency components correspond to rough textures and edges
  - Low frequency components correspond to smooth regions
- Fourier coefficients are complex (with real and imaginary parts)
  - Can also be decomposed into amplitude and phase
  - Somehow phases carry more semantic information
- Can “filter” an image by removing some frequency components of an image
  - Low-pass filter (removing HP) tends to smooth out an image
- Images usually compressed more efficiently in frequency domain (e.g. JPEG)