

Neural Networks

Deep Learning Lecture 3

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- Need your presentation preference by the end of this class. Again, please give me three package names with order of preference. The finally decision will be computed by minimizing the following cost function :)

- $\sum_{\text{student}} \text{student cost} + \sum_{\text{package}} \text{package cost}$

- $\text{student cost} = \begin{cases} 0, & \text{first priority} \\ 2.5, & \text{second priority} \\ 5, & \text{third priority} \end{cases}$

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HW1 due next week

Review

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- Basic concepts of regression and classification
- Examples of regularization such as ridge (l_2) regression and lasso (l_1)
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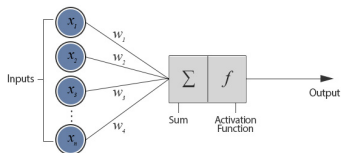
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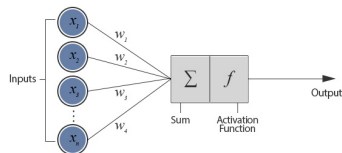
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 - We introduced loss functions and the concept of training a classifier through minimizing the loss function
 - We described stochastic gradient descent and momentum trick for classification

Perceptron

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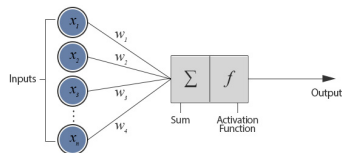


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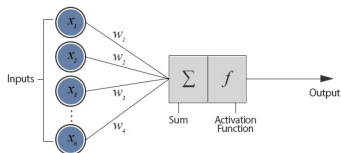
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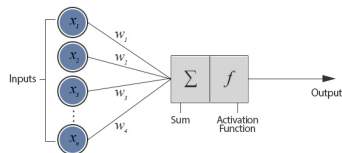
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- Perceptrons are still used in systems with large number (millions) of features. Other than that, it has relatively limited use since most problems are not linearly separable

Perceptron

- In most cases, perceptron would be useful if only one manages to handcode inputs into separable features

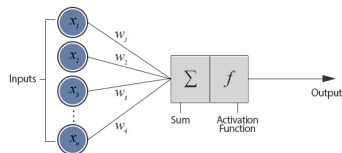


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- That was the main area of research in many machine learning applications—finding efficient ways to generate good features

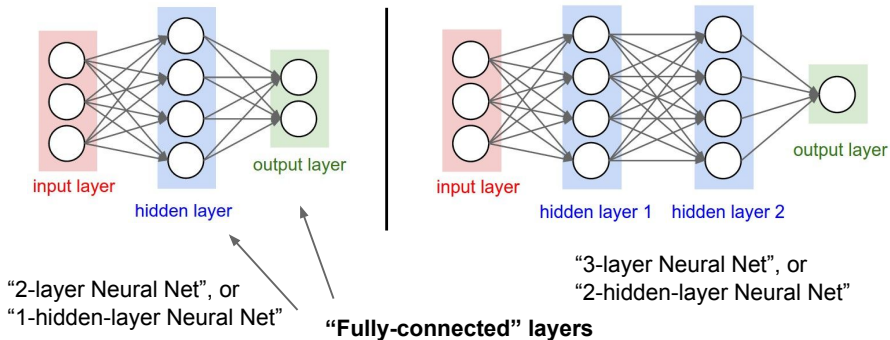
Perceptron



- In most cases, perceptron would be useful if only one manages to handcode inputs into separable features
- That was the main area of research in many machine learning applications—finding efficient ways to generate good features
- One attractive characteristic of deep learning (neural networks) is that we not only can train the classifier but also can learn the appropriate features

Nomenclature of basic network architectures

Neural Networks: Architectures



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 - Neuron no longer will be restricted to summation and activation function but can be any computation as well (e.g., max)

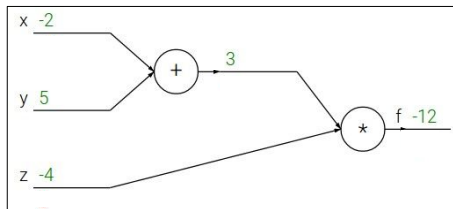
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- Let me try to explain through an example

A simple BP example

$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$



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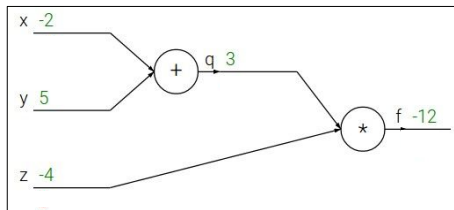
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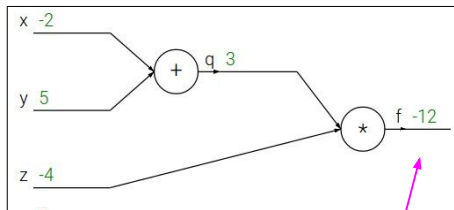
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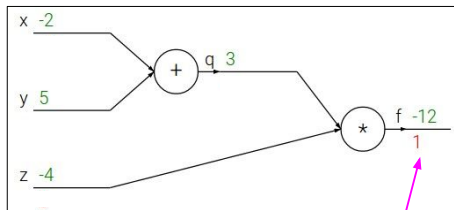
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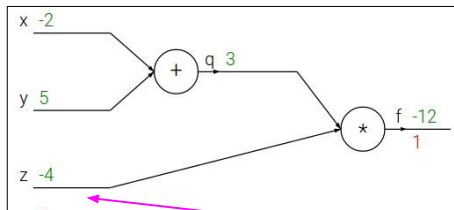
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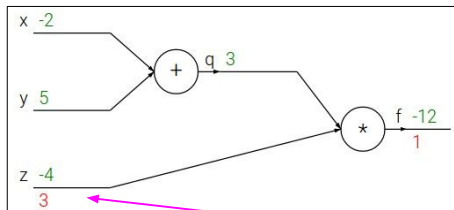
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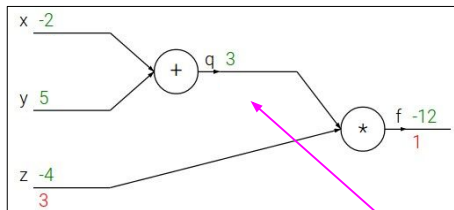
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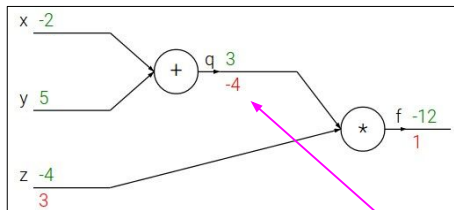
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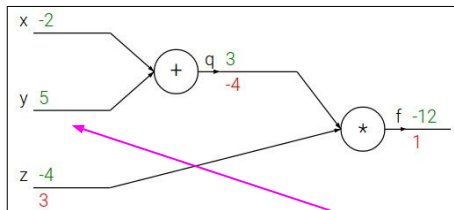
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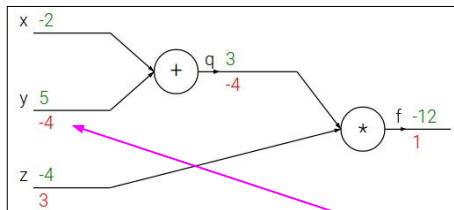
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Chain rule:

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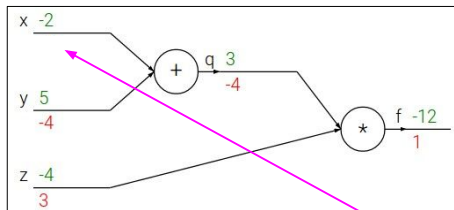
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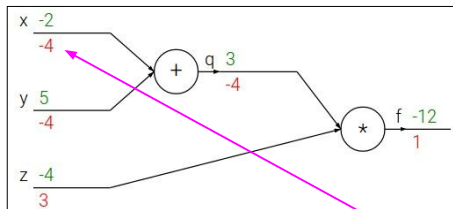
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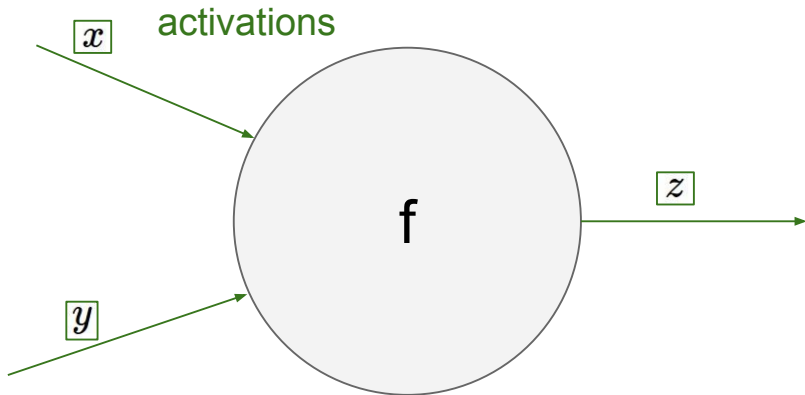


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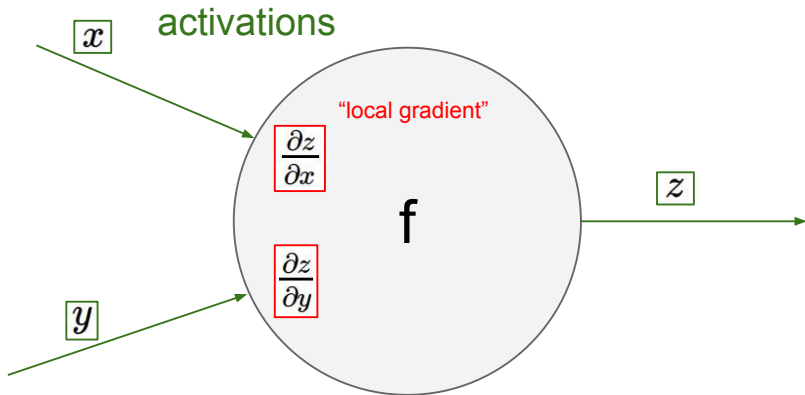
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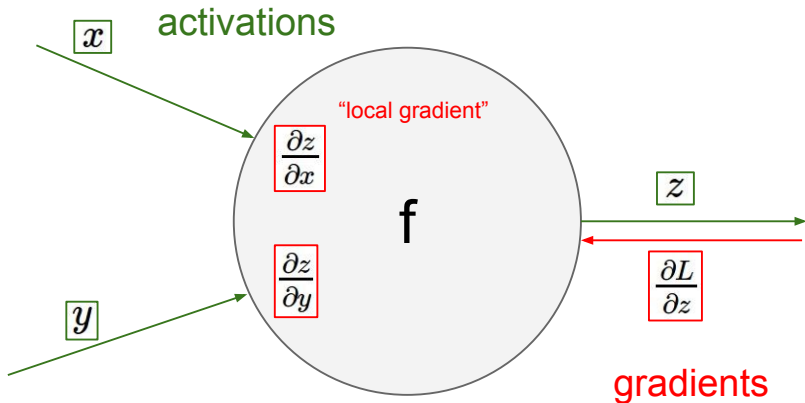
BP at one node



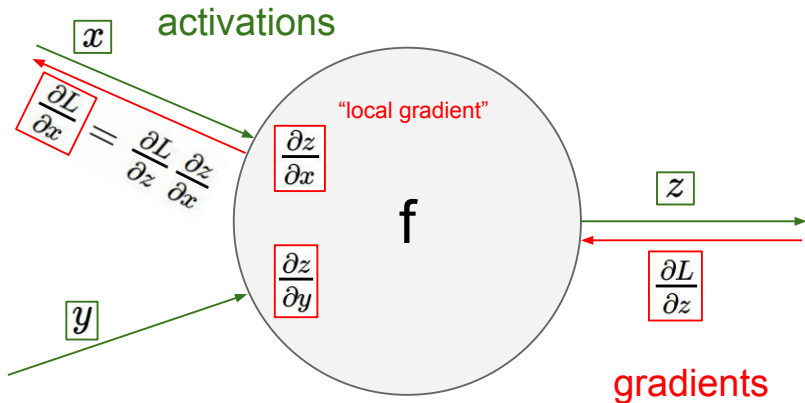
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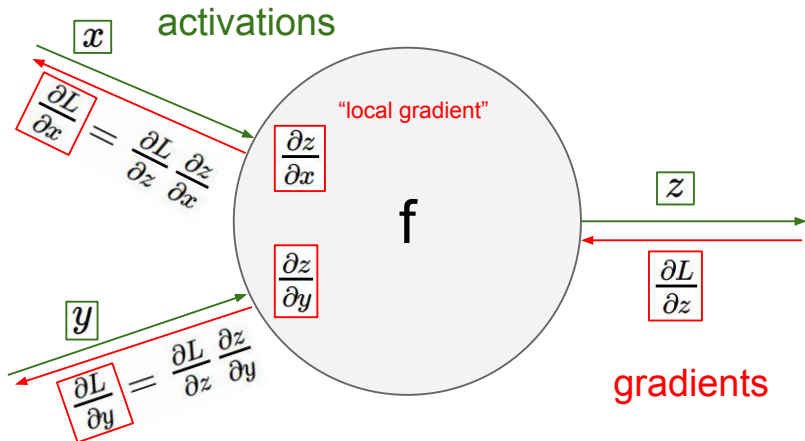
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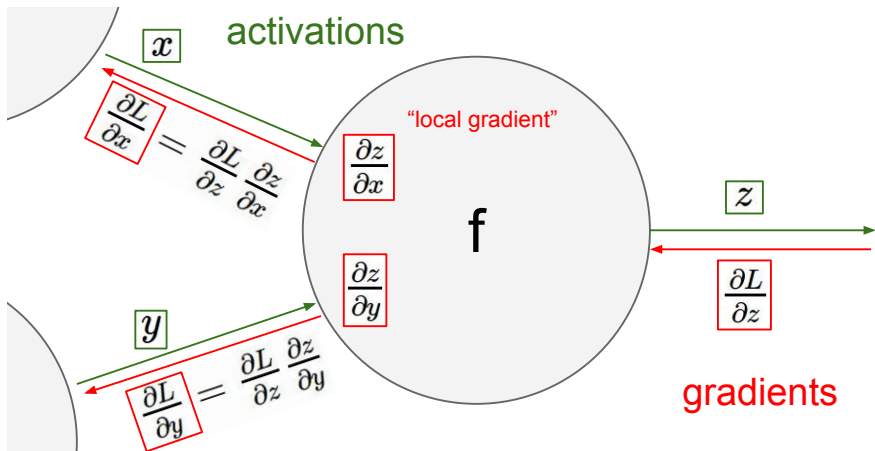
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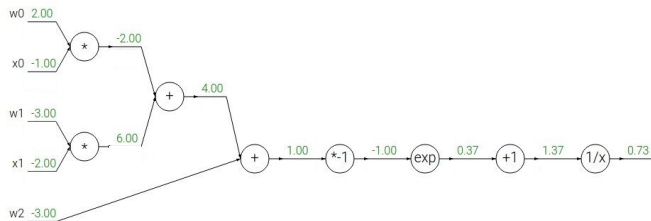


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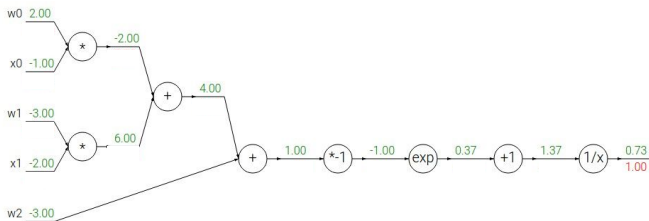
Yet another BP example

Another example: $f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$



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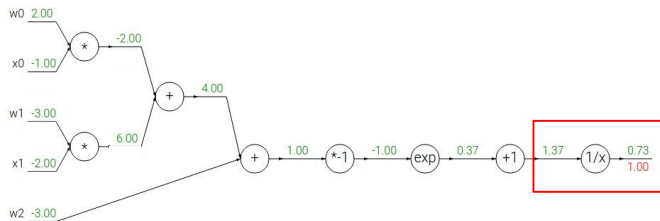
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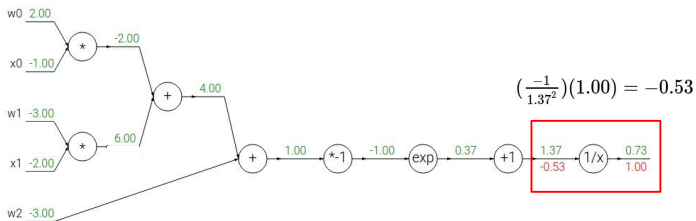
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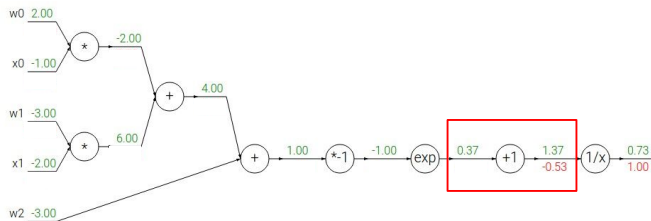
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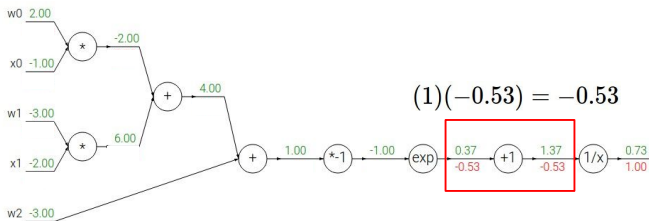
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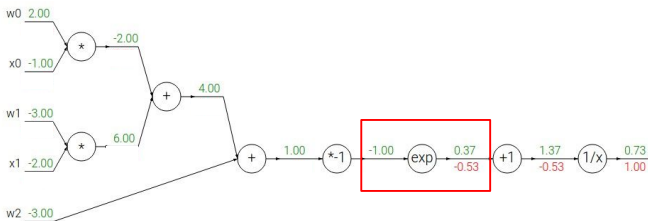
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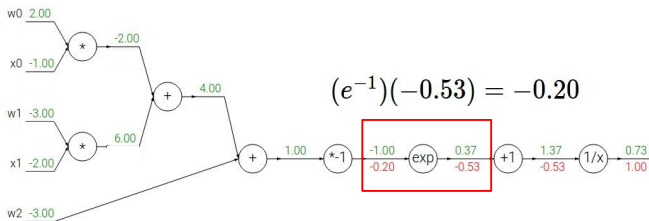
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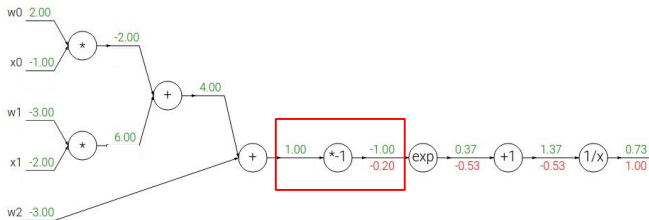
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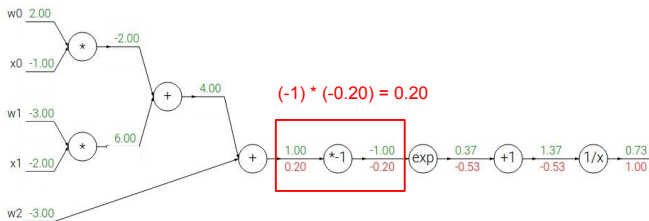
$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Yet another BP example

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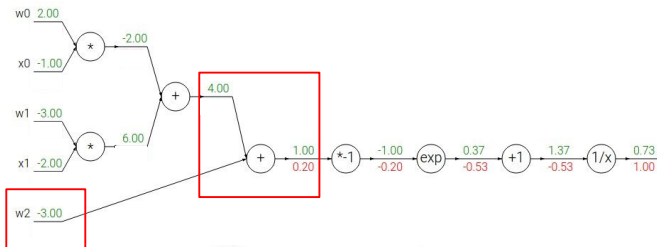
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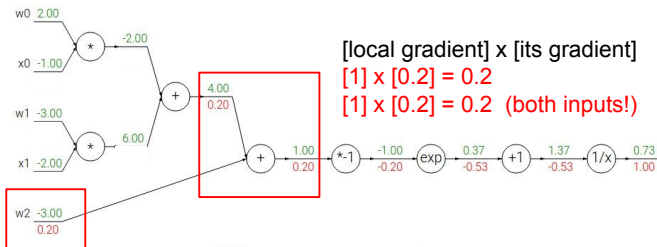
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$$\begin{array}{lcl}
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 \end{array}
 \quad \Bigg| \quad
 \begin{array}{lcl}
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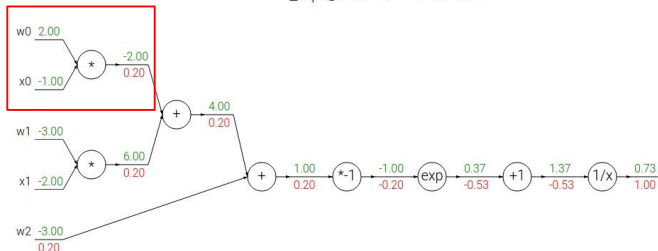
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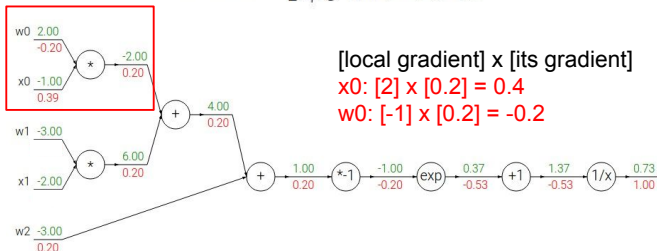
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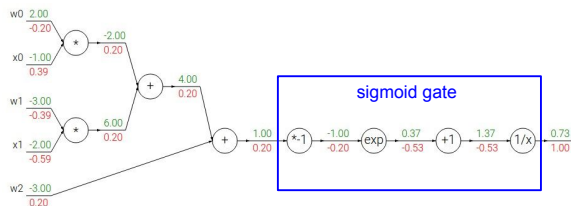
Yet another BP example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$



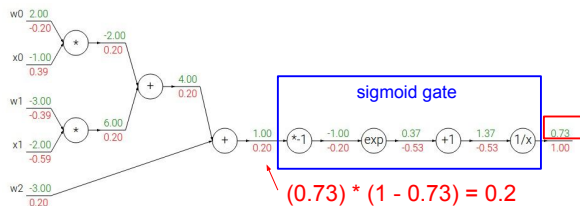
Yet another BP example

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sigmoid function

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Remark of BP

- During the **forward** pass, each computing unit will evaluate the output and also the corresponding local derivatives of the output w.r.t. the inputs

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Remark of BP

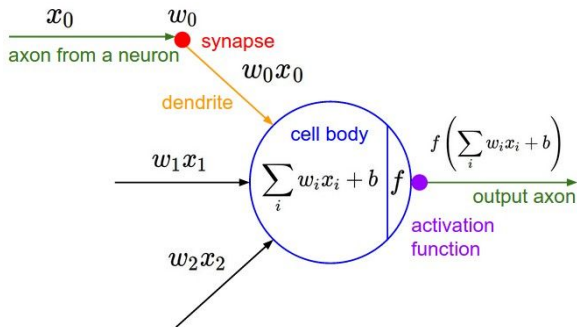
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 - For a large network, there can be a large spike of memory consumption during the forward pass

Remark of BP

- During the **forward** pass, each computing unit will evaluate the output and also the corresponding local derivatives of the output w.r.t. the inputs
- During the **backward** pass, the local derivatives and the evaluated outputs will be “consumed” to compute the overall derivatives
 - For a large network, there can be a large spike of memory consumption during the forward pass
- Note that BP only computes the gradients. It does not perform the optimization. Sometimes you may hear people said that they trained their networks with BP. What they said was not literally right. We will discuss more on optimizer later today

Activation functions

Activation Functions

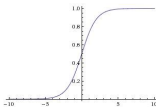


Activation functions

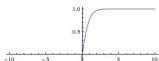
Activation Functions

Sigmoid

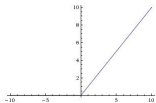
$$\sigma(x) = 1/(1 + e^{-x})$$



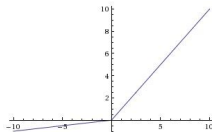
tanh tanh(x)



ReLU max(0,x)



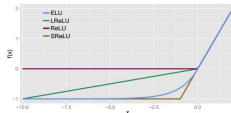
Leaky ReLU max(0.1x, x)



Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

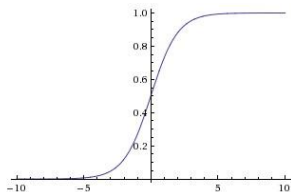
ELU

$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha(\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$



Activation functions

Activation Functions



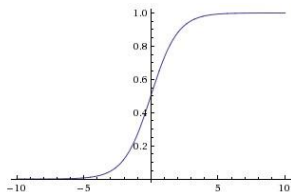
Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

Activation functions

Activation Functions



Sigmoid

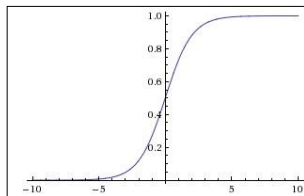
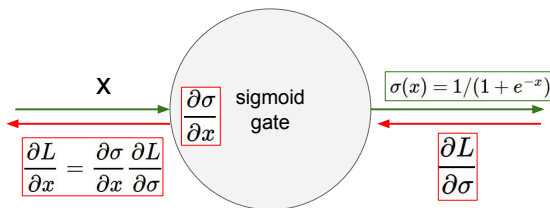
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3 problems:

1. Saturated neurons “kill” the gradients

Activation functions



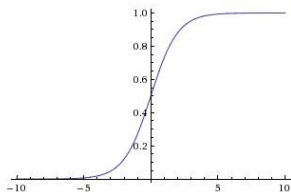
What happens when $x = -10$?

What happens when $x = 0$?

What happens when $x = 10$?

Activation functions

Activation Functions



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

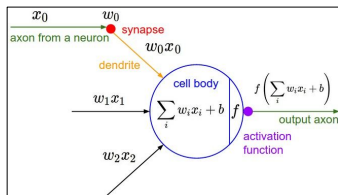
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3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Activation functions

Consider what happens when the input to a neuron (x) is always positive:



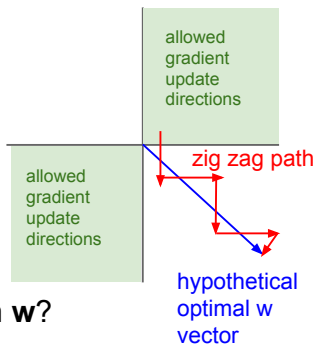
$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on \mathbf{w} ?

Activation functions

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

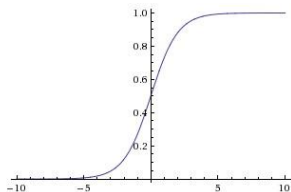


What can we say about the gradients on w ?

Always all positive or all negative :(
(this is also why you want zero-mean data!)

Activation functions

Activation Functions



Sigmoid

$$\sigma(x) = 1/(1 + e^{-x})$$

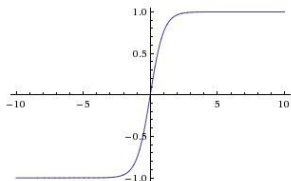
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- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

3 problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered
3. $\exp()$ is a bit compute expensive

Activation functions

Activation Functions



$\tanh(x)$

- Squashes numbers to range $[-1,1]$
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

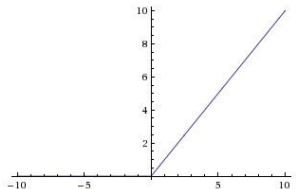
Fei-Fei Li & Andrej Karpathy & Justin Johnson

Lecture 5 - 37

20 Jan 2016

Activation functions

Activation Functions



- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

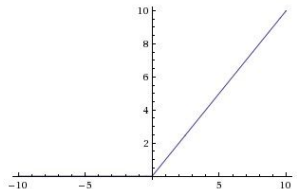
ReLU

(Rectified Linear Unit)

[Krizhevsky et al., 2012]

Activation functions

Activation Functions



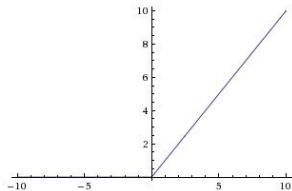
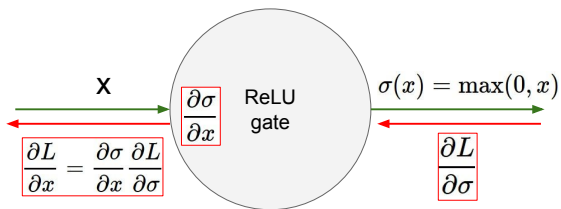
ReLU

(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)
- Not zero-centered output
- An annoyance:

hint: what is the gradient when $x < 0$?

Activation functions

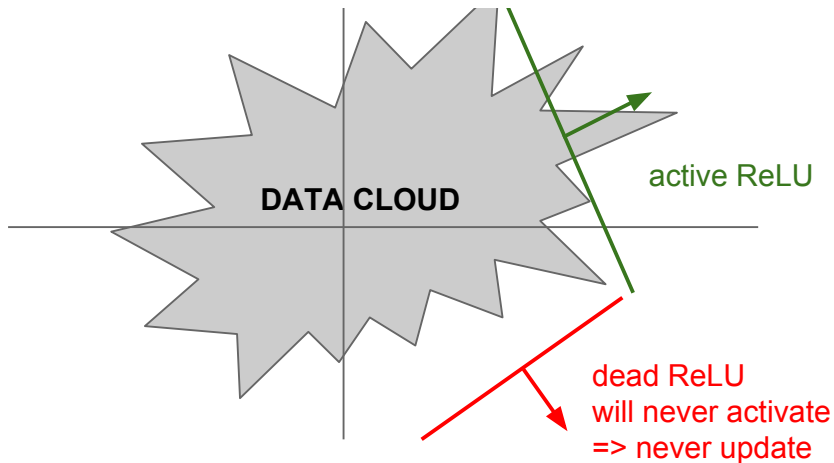


What happens when $x = -10$?

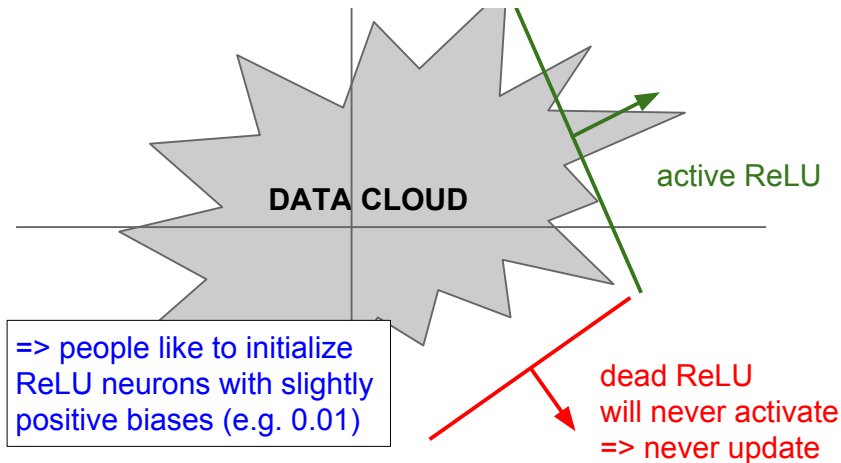
What happens when $x = 0$?

What happens when $x = 10$?

Activation functions



Activation functions

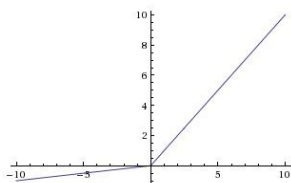


Activation functions

Activation Functions

[Mass et al., 2013]

[He et al., 2015]



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”**.

Leaky ReLU

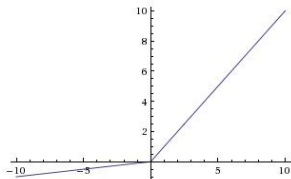
$$f(x) = \max(0.01x, x)$$

Activation functions

Activation Functions

[Mass et al., 2013]

[He et al., 2015]



Leaky ReLU

$$f(x) = \max(0.01x, x)$$

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

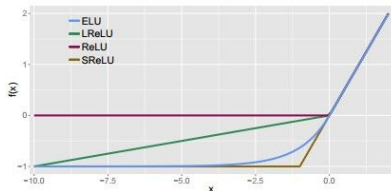
backprop into α
(parameter)

Activation functions

Activation Functions

[Clevert et al., 2015]

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

- All benefits of ReLU
- Does not die
- Closer to zero mean outputs
- Computation requires $\exp()$

Activation functions

Maxout "Neurons" [Goodfellow et al., 2013]

- Try to generalize ReLU and leaky ReLU

$$\max(\mathbf{w}_1^T \mathbf{x} + b_1, \mathbf{w}_2^T \mathbf{x} + b_2)$$

Activation functions

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- Try to generalize ReLU and leaky ReLU

$$\max(\mathbf{w}_1^T \mathbf{x} + b_1, \mathbf{w}_2^T \mathbf{x} + b_2)$$

Pros

- Linear regime
- Does not saturate
- Does not die

Activation functions

Maxout "Neurons" [Goodfellow et al., 2013]

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$$\max(\mathbf{w}_1^T \mathbf{x} + b_1, \mathbf{w}_2^T \mathbf{x} + b_2)$$

Pros

- Linear regime
- Does not saturate
- Does not die

Cons

- Double amount of parameters

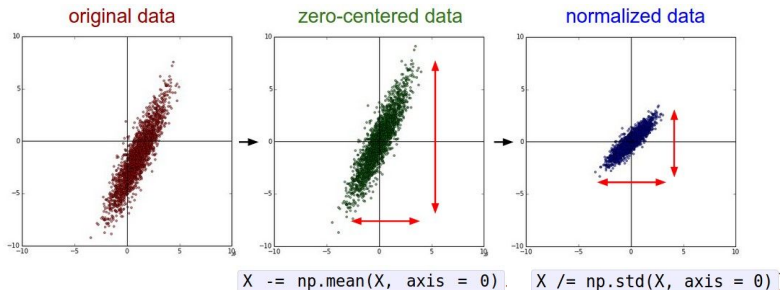
Activation functions

TLDR: In practice:

- Use **ReLU**. Be careful with your learning rates
- Try out **Leaky ReLU / Maxout / ELU**
- Try out **tanh** but don't expect much
- **Don't use sigmoid**

Input preprocessing

Step 1: Preprocess the data

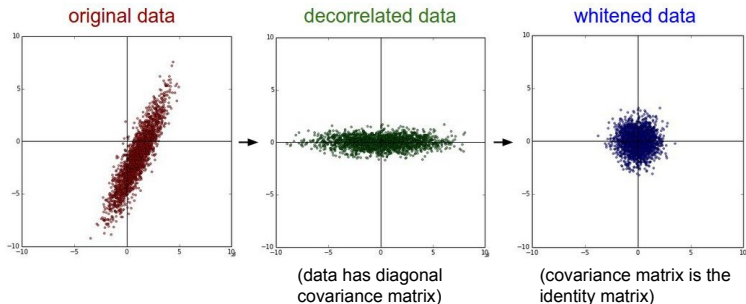


(Assume X [NxD] is data matrix,
each example in a row)

Input preprocessing

Step 1: Preprocess the data

In practice, you may also see **PCA** and **Whitening** of the data



Input preprocessing

TLDR: In practice for Images: center only

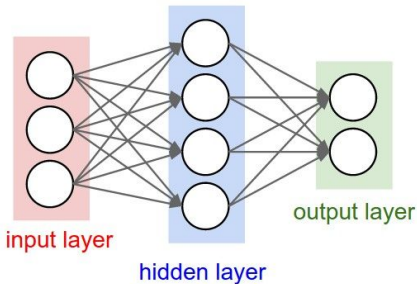
e.g. consider CIFAR-10 example with [32,32,3] images

- Subtract the mean image (e.g. AlexNet)
(mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)
(mean along each channel = 3 numbers)

Not common to normalize
variance, to do PCA or
whitening

Weight initialization

- Q: what happens when $W=0$ init is used?



Weight initialization

- First idea: **Small random numbers**
(gaussian with zero mean and $1e-2$ standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

Weight initialization

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(gaussian with zero mean and $1e-2$ standard deviation)

```
W = 0.01* np.random.randn(D,H)
```

Works ~okay for small networks, but can lead to non-homogeneous distributions of activations across the layers of a network.

Weight initialization

Lets look at some activation statistics

E.g. 10-layer net with 500 neurons on each layer, using tanh nonlinearities, and initializing as described in last slide.

```
# assume some unit gaussian 10-D input data
D = np.random.randn(1000, 500)
hidden_layer_sizes = [500]*10
nonlinearities = ['tanh']*len(hidden_layer_sizes)

act = {'relu':lambda x:np.maximum(0,x), 'tanh':lambda x:np.tanh(x)}
Hs = {}
for i in xrange(len(hidden_layer_sizes)):
    X = D if i == 0 else Hs[i-1] # input at this layer
    fan_in = X.shape[1]
    fan_out = hidden_layer_sizes[i]
    W = np.random.randn(fan_in, fan_out) * 0.01 # layer initialization

    H = np.dot(X, W) # matrix multiply
    H = act[nonlinearities[i]](H) # nonlinearity
    Hs[i] = H # cache result on this layer

# look at distributions at each layer
print 'input layer had mean %f and std %f' % (np.mean(D), np.std(D))
layer_means = [np.mean(H) for i,H in Hs.iteritems()]
layer_stds = [np.std(H) for i,H in Hs.iteritems()]
for i,H in Hs.iteritems():
    print 'hidden layer %d had mean %f and std %f' % (i+1, layer_means[i], layer_stds[i])

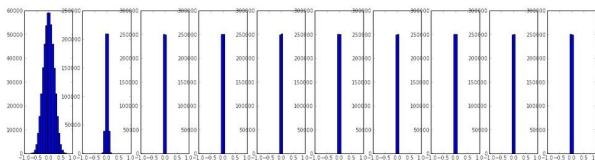
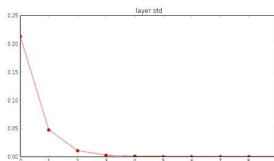
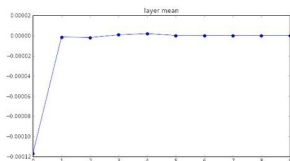
# plot the means and standard deviations
plt.figure()
plt.subplot(121)
plt.plot(Hs.keys(), layer_means, 'ob-')
plt.title('layer mean')
plt.subplot(122)
plt.plot(Hs.keys(), layer_stds, 'or-')
plt.title('layer std')

# plot the raw distributions
plt.figure()
for i,H in Hs.iteritems():
    plt.subplot(1,len(Hs),i+1)
    plt.hist(H.ravel(), 30, range=(-1,1))
```

Weight initialization

```

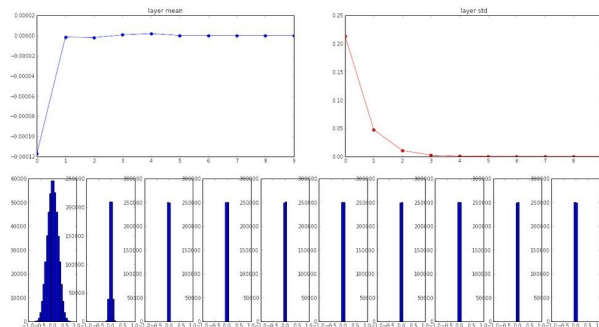
input layer had mean 0.000927 and std 0.998388
hidden layer 1 had mean -0.000117 and std 0.213081
hidden layer 2 had mean -0.000001 and std 0.047551
hidden layer 3 had mean -0.000002 and std 0.010630
hidden layer 4 had mean 0.000001 and std 0.002378
hidden layer 5 had mean 0.000002 and std 0.000532
hidden layer 6 had mean -0.000000 and std 0.000119
hidden layer 7 had mean 0.000000 and std 0.000026
hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
  
```



Weight initialization

```

input layer had mean 0.000927 and std 0.998388
hidden layer 1 had mean -0.000117 and std 0.213081
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hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
  
```



All activations become zero!

Q: think about the backward pass. What do the gradients look like?

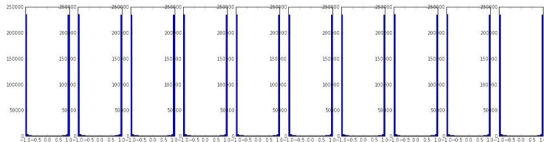
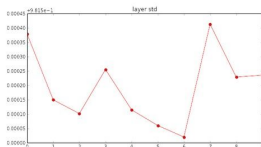
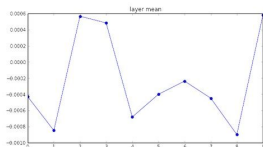
Hint: think about backward pass for a $W \cdot X$ gate.

Weight initialization

```
W = np.random.randn(fan_in, fan_out) * 1.0 # layer initialization
```

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean -0.000430 and std 0.981879
hidden layer 2 had mean -0.000849 and std 0.981649
hidden layer 3 had mean 0.000566 and std 0.981601
hidden layer 4 had mean 0.000483 and std 0.981755
hidden layer 5 had mean -0.000682 and std 0.981614
hidden layer 6 had mean -0.000401 and std 0.981560
hidden layer 7 had mean -0.000237 and std 0.981520
hidden layer 8 had mean -0.000448 and std 0.981913
hidden layer 9 had mean -0.000899 and std 0.981728
hidden layer 10 had mean 0.000584 and std 0.981736
```

*1.0 instead of *0.01



Almost all neurons completely saturated, either -1 and 1. Gradients will be all zero.

Variance calibration for linear layer

Assume linear activation and zero-mean weights and inputs. And number of inputs is n . Then,

$$\text{Var}(y) = \text{Var} \left(\sum_i^n w_i x_i \right) = \sum_i^n \text{Var}(w_i x_i)$$

Variance calibration for linear layer

Assume linear activation and zero-mean weights and inputs. And number of inputs is n . Then,

$$\begin{aligned}\text{Var}(y) &= \text{Var}\left(\sum_i^n w_i x_i\right) = \sum_i^n \text{Var}(w_i x_i) \\ &= \sum_i^n [E(w_i)]^2 \text{Var}(x_i) + E[(x_i)]^2 \text{Var}(w_i) + \text{Var}(x_i) \text{Var}(w_i)\end{aligned}$$

Variance calibration for linear layer

Assume linear activation and zero-mean weights and inputs. And number of inputs is n . Then,

$$\begin{aligned}\text{Var}(y) &= \text{Var}\left(\sum_i^n w_i x_i\right) = \sum_i^n \text{Var}(w_i x_i) \\ &= \sum_i^n [E(w_i)]^2 \text{Var}(x_i) + E[(x_i)]^2 \text{Var}(w_i) + \text{Var}(x_i) \text{Var}(w_i) \\ &= \sum_i^n \text{Var}(x_i) \text{Var}(w_i) \\ &= (n \text{Var}(w)) \text{Var}(x)\end{aligned}$$

Variance calibration for linear layer

Assume linear activation and zero-mean weights and inputs. And number of inputs is n . Then,

$$\begin{aligned}\text{Var}(y) &= \text{Var}\left(\sum_i^n w_i x_i\right) = \sum_i^n \text{Var}(w_i x_i) \\ &= \sum_i^n [E(w_i)]^2 \text{Var}(x_i) + E[(x_i)]^2 \text{Var}(w_i) + \text{Var}(x_i) \text{Var}(w_i) \\ &= \sum_i^n \text{Var}(x_i) \text{Var}(w_i) \\ &= (n \text{Var}(w)) \text{Var}(x)\end{aligned}$$

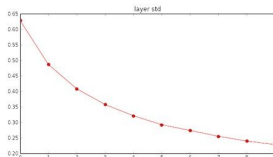
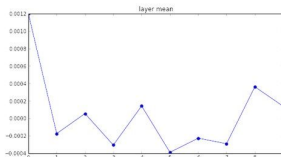
Thus, output will have same variance as input if $n \text{Var}(w) = 1$

Weight initialization

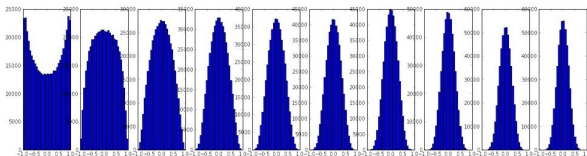
input layer had mean 0.001800 and std 1.001311
 hidden layer 1 had mean 0.001198 and std 0.627953
 hidden layer 2 had mean -0.000175 and std 0.486051
 hidden layer 3 had mean 0.000055 and std 0.407723
 hidden layer 4 had mean -0.000306 and std 0.357108
 hidden layer 5 had mean 0.000142 and std 0.320917
 hidden layer 6 had mean -0.000389 and std 0.292116
 hidden layer 7 had mean -0.000228 and std 0.273387
 hidden layer 8 had mean -0.000291 and std 0.254935
 hidden layer 9 had mean 0.000361 and std 0.239266
 hidden layer 10 had mean 0.000139 and std 0.228008

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

“Xavier initialization”
 [Glorot et al., 2010]



Reasonable initialization.
 (Mathematical derivation
 assumes linear activations)

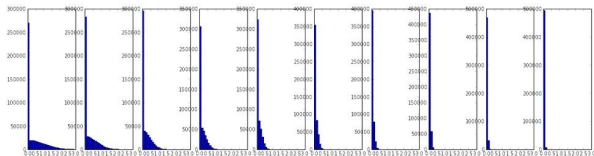
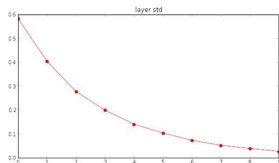
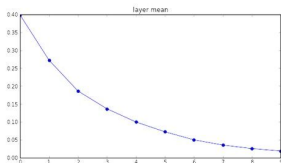


Weight initialization

input layer had mean 0.009501 and std 0.999444
 hidden layer 1 had mean 0.398623 and std 0.582273
 hidden layer 2 had mean 0.272352 and std 0.403795
 hidden layer 3 had mean 0.186076 and std 0.276912
 hidden layer 4 had mean 0.136442 and std 0.198685
 hidden layer 5 had mean 0.099568 and std 0.140299
 hidden layer 6 had mean 0.072234 and std 0.103280
 hidden layer 7 had mean 0.049775 and std 0.072748
 hidden layer 8 had mean 0.035138 and std 0.051572
 hidden layer 9 had mean 0.025404 and std 0.038583
 hidden layer 10 had mean 0.018408 and std 0.026076

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

but when using the ReLU nonlinearity it breaks.



Variance calibration for ReLU

Note that it doesn't work when the activation layer is ReLU. But...¹

$$\begin{aligned}\text{Var}(y^{(l)}) &= \text{Var}\left(\sum_i^n w_i^{(l)} x_i^{(l)}\right) = \sum_i^n \text{Var}(w_i^{(l)} x_i^{(l)}) = n \text{Var}(w^{(l)} x^{(l)}) \\ &= n E(w^{(l)})^2 \text{Var}(x^{(l)}) + n E(x^{(l)})^2 \text{Var}(w^{(l)}) + n \text{Var}(x^{(l)}) \text{Var}(w^{(l)})\end{aligned}$$

¹Note that $y^{(l)}$ now denotes the sum of input before going through the activation function.

Variance calibration for ReLU

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$$\begin{aligned}\text{Var}(y^{(l)}) &= \text{Var}\left(\sum_i^n w_i^{(l)} x_i^{(l)}\right) = \sum_i^n \text{Var}(w_i^{(l)} x_i^{(l)}) = n \text{Var}(w^{(l)} x^{(l)}) \\ &= n E(w^{(l)})^2 \text{Var}(x^{(l)}) + n E(x^{(l)})^2 \text{Var}(w^{(l)}) + n \text{Var}(x^{(l)}) \text{Var}(w^{(l)}) \\ &= n E(x^{(l)})^2 \text{Var}(w^{(l)}) + n \text{Var}(x^{(l)}) \text{Var}(w^{(l)})\end{aligned}$$

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¹Note that $y^{(l)}$ now denotes the sum of input before going through the activation function.

Variance calibration for ReLU

Note that it doesn't work when the activation layer is ReLU. But...¹

$$\begin{aligned}
 \text{Var}(y^{(l)}) &= \text{Var}\left(\sum_i^n w_i^{(l)} x_i^{(l)}\right) = \sum_i^n \text{Var}(w_i^{(l)} x_i^{(l)}) = n \text{Var}(w^{(l)} x^{(l)}) \\
 &= n E(w^{(l)})^2 \text{Var}(x^{(l)}) + n E(x^{(l)})^2 \text{Var}(w^{(l)}) + n \text{Var}(x^{(l)}) \text{Var}(w^{(l)}) \\
 &= n E(x^{(l)})^2 \text{Var}(w^{(l)}) + n \text{Var}(x^{(l)}) \text{Var}(w^{(l)}) \\
 &= n E((x^{(l)})^2) \text{Var}(w^{(l)}) \\
 &= n [\text{Var}(y^{(l-1)})/2] \text{Var}(w^{(l)}) = \left[\frac{n}{2} \text{Var}(w^{(l)})\right] \text{Var}(y^{(l-1)})
 \end{aligned}$$

Variance of y conserved across a layer if $\frac{n}{2} \text{Var}(w) = 1$

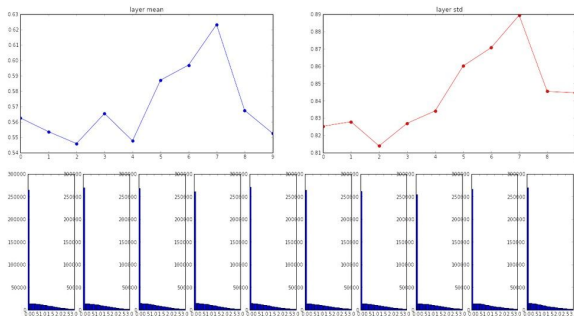
¹Note that $y^{(l)}$ now denotes the sum of input before going through the activation function.

Weight initialization

input layer had mean 0.000501 and std 0.999444
 hidden layer 1 had mean 0.562488 and std 0.825232
 hidden layer 2 had mean 0.553614 and std 0.827835
 hidden layer 3 had mean 0.545867 and std 0.813855
 hidden layer 4 had mean 0.565396 and std 0.826902
 hidden layer 5 had mean 0.547678 and std 0.834092
 hidden layer 6 had mean 0.587103 and std 0.860035
 hidden layer 7 had mean 0.596867 and std 0.870610
 hidden layer 8 had mean 0.623214 and std 0.809340
 hidden layer 9 had mean 0.567490 and std 0.845357
 hidden layer 10 had mean 0.552531 and std 0.844523

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015
 (note additional /2)

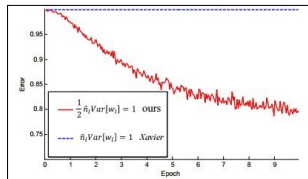
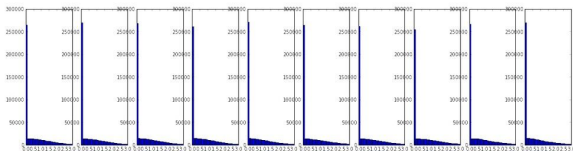
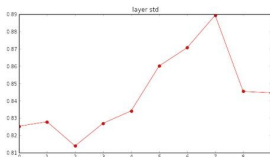
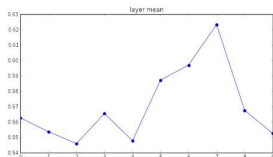


Weight initialization

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```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015
 (note additional /2)



Weight initialization

Proper initialization is an active area of research...

Understanding the difficulty of training deep feedforward neural networks

by Glorot and Bengio, 2010

Exact solutions to the nonlinear dynamics of learning in deep linear neural networks by

Saxe et al, 2013

Random walk initialization for training very deep feedforward networks by Sussillo and

Abbott, 2014

Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification by He et al., 2015

Data-dependent Initializations of Convolutional Neural Networks by Krähenbühl et al., 2015

All you need is a good init, Mishkin and Matas, 2015

...

Fei-Fei Li & Andrej Karpathy & Justin Johnson

Lecture 5 - 64

20 Jan 2016

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want unit gaussian activations? just make them so.”

consider a batch of activations at some layer.
To make each dimension unit gaussian, apply:

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

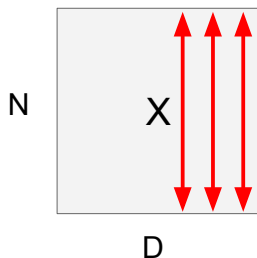
this is a vanilla
differentiable function...

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

“you want unit gaussian activations?
just make them so.”



1. compute the empirical mean and variance independently for each dimension.

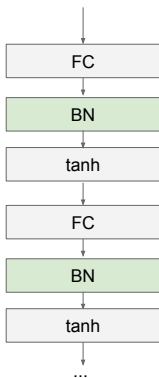
2. Normalize

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]



Usually inserted after Fully Connected / (or Convolutional, as we'll see soon) layers, and before nonlinearity.

Problem: do we necessarily want a unit gaussian input to a tanh layer?

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

Normalize:

$$\hat{x}^{(k)} = \frac{x^{(k)} - \mathbb{E}[x^{(k)}]}{\sqrt{\text{Var}[x^{(k)}]}}$$

And then allow the network to squash the range if it wants to:

$$y^{(k)} = \gamma^{(k)} \hat{x}^{(k)} + \beta^{(k)}$$

Note, the network can learn:

$$\gamma^{(k)} = \sqrt{\text{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathbb{E}[x^{(k)}]$$

to recover the identity mapping.

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma, \beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m} \sum_{i=1}^m x_i \quad // \text{ mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m} \sum_{i=1}^m (x_i - \mu_{\mathcal{B}})^2 \quad // \text{ mini-batch variance}$$

$$\hat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \quad // \text{ normalize}$$

$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma, \beta}(x_i) \quad // \text{ scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe

Batch normalization

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\dots m}\}$;
Parameters to be learned: γ, β

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$$y_i \leftarrow \gamma \hat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i) \quad // \text{ scale and shift}$$

Note: at test time BatchNorm layer functions differently:

The mean/std are not computed based on the batch. Instead, a single fixed empirical mean of activations during training is used.

(e.g. can be estimated during training with running averages)

Ensemble trick

1. Train multiple independent models
2. At test time average their results

Enjoy 2% extra performance

Ensemble trick

Fun Tips/Tricks:

- can also get a small boost from averaging multiple model checkpoints of a single model.

Ensemble trick

Fun Tips/Tricks:

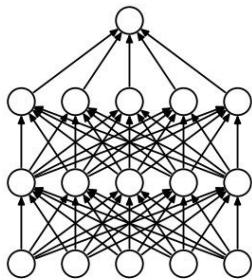
- can also get a small boost from averaging multiple model checkpoints of a single model.
- keep track of (and use at test time) a running average parameter vector:

```
while True:
    data_batch = dataset.sample_data_batch()
    loss = network.forward(data_batch)
    dx = network.backward()
    x += - learning_rate * dx
    x_test = 0.995*x_test + 0.005*x # use for test set
```

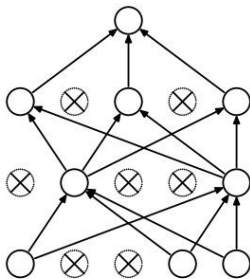
Dropout

Regularization: **Dropout**

“randomly set some neurons to zero in the forward pass”



(a) Standard Neural Net



(b) After applying dropout.

[Srivastava et al., 2014]

Dropout

```
p = 0.5 # probability of keeping a unit active. higher = less dropout
```

```
def train_step(X):
```

```
    """ X contains the data """
```

```
    # forward pass for example 3-layer neural network
```

```
    H1 = np.maximum(0, np.dot(W1, X) + b1)
```

```
    U1 = np.random.rand(*H1.shape) < p # first dropout mask
```

```
    H1 *= U1 # drop!
```

```
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
```

```
    U2 = np.random.rand(*H2.shape) < p # second dropout mask
```

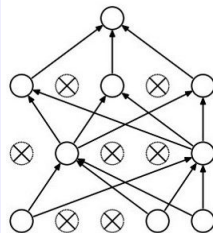
```
    H2 *= U2 # drop!
```

```
    out = np.dot(W3, H2) + b3
```

```
    # backward pass: compute gradients... (not shown)
```

```
    # perform parameter update... (not shown)
```

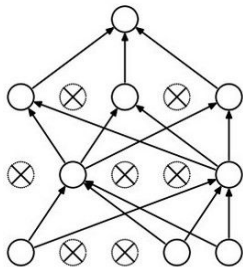
Example forward pass with a 3-layer network using dropout



Dropout

Waaaait a second...

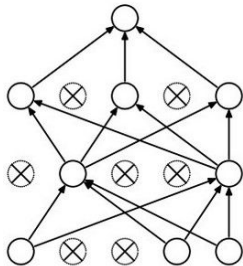
How could this possibly be a good idea?



Dropout

Waaaait a second...

How could this possibly be a good idea?



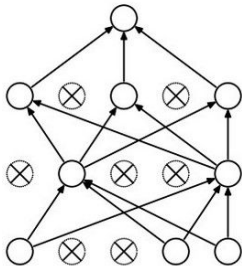
Forces the network to have a redundant representation.



Dropout

Waaaait a second...

How could this possibly be a good idea?



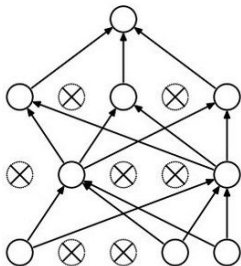
Another interpretation:

Dropout is training a large ensemble of models (that share parameters).

Each binary mask is one model, gets trained on only \sim one datapoint.

Dropout

At test time....



Ideally:

want to integrate out all the noise

Monte Carlo approximation:

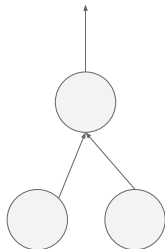
do many forward passes with different dropout masks, average all predictions

Dropout

At test time....

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).



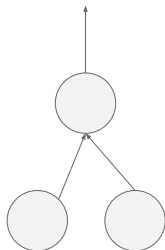
(this can be shown to be an approximation to evaluating the whole ensemble)

Dropout

At test time....

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).



Q: Suppose that with all inputs present at test time the output of this neuron is x .

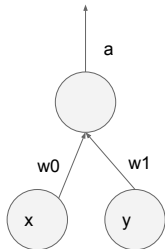
What would its output be during training time, in expectation? (e.g. if $p = 0.5$)

Dropout

At test time....

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).



during test: $\mathbf{a = w0*x + w1*y}$

during train:

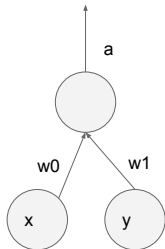
$$\begin{aligned}
 E[a] &= \frac{1}{4} * (w0*0 + w1*0 \\
 &\quad w0*0 + w1*y \\
 &\quad w0*x + w1*0 \\
 &\quad w0*x + w1*y) \\
 &= \frac{1}{4} * (2 w0*x + 2 w1*y) \\
 &= \frac{1}{2} * (\mathbf{w0*x + w1*y})
 \end{aligned}$$

Dropout

At test time....

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).



during test: $\mathbf{a = w0*x + w1*y}$

during train:

$$\begin{aligned}
 E[a] &= \frac{1}{4} * (w0*0 + w1*0 \\
 &\quad w0*0 + w1*y \\
 &\quad w0*x + w1*0 \\
 &\quad w0*x + w1*y) \\
 &= \frac{1}{4} * (2 w0*x + 2 w1*y) \\
 &= \frac{1}{2} * (w0*x + w1*y)
 \end{aligned}$$

With $p=0.5$, using all inputs in the forward pass would inflate the activations by 2x from what the network was "used to" during training!
 \Rightarrow Have to compensate by scaling the activations back down by $\frac{1}{2}$

Dropout

We can do something approximate analytically

```
def predict(X):  
    # ensembled forward pass  
    H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations  
    H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations  
    out = np.dot(W3, H2) + b3
```

At test time all neurons are active always

=> We must scale the activations so that for each neuron:
output at test time = expected output at training time

Dropout

```

""" Vanilla Dropout: Not recommended implementation (see notes below) """
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
    """ X contains the data """

    # forward pass for example 3-layer neural network
    H1 = np.maximum(0, np.dot(W1, X) + b1)
    U1 = np.random.rand(*H1.shape) < p # first dropout mask
    H1 *= U1 # drop!
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    U2 = np.random.rand(*H2.shape) < p # second dropout mask
    H2 *= U2 # drop!
    out = np.dot(W3, H2) + b3

    # backward pass: compute gradients... (not shown)
    # perform parameter update... (not shown)

def predict(X):
    # ensemble forward pass
    H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
    H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
    out = np.dot(W3, H2) + b3

```

Dropout Summary

drop in forward pass

scale at test time

Dropout

More common: “Inverted dropout”

```

p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
    # forward pass for example 3-layer neural network
    H1 = np.maximum(0, np.dot(W1, X) + b1)
    U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
    H1 *= U1 # drop!
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask. Notice /p!
    H2 *= U2 # drop!
    out = np.dot(W3, H2) + b3

    # backward pass: compute gradients... (not shown)
    # perform parameter update... (not shown)

def predict(X):
    # ensemble forward pass
    H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
    H2 = np.maximum(0, np.dot(W2, H1) + b2)
    out = np.dot(W3, H2) + b3
  
```

test time is unchanged!



Optimizers

Training a neural network, main loop:

```
while True:
    data_batch = dataset.sample_data_batch()
    loss = network.forward(data_batch)
    dx = network.backward()
    x += - learning_rate * dx
```

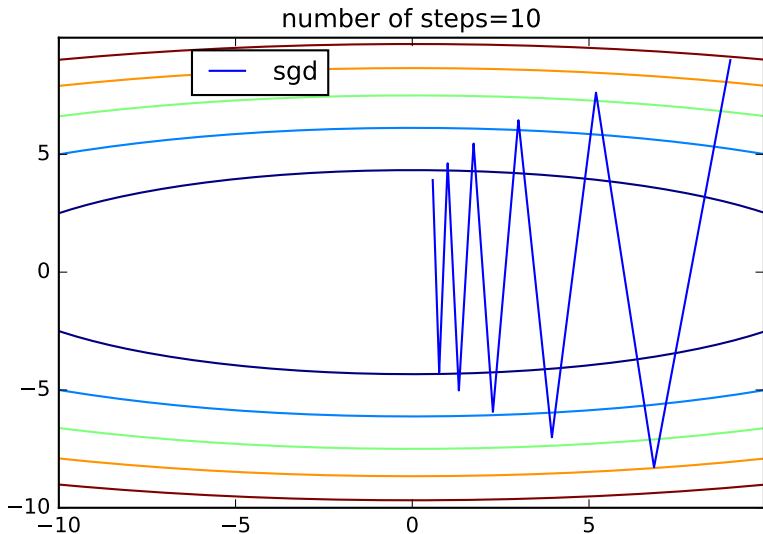
Optimizers

Training a neural network, main loop:

```
while True:
    data_batch = dataset.sample_data_batch()
    loss = network.forward(data_batch)
    dx = network.backward()
    x += - learning_rate * dx
```

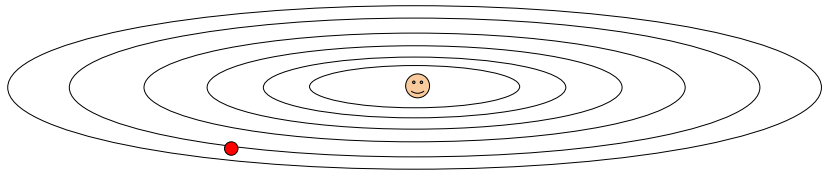
simple gradient descent update
now: complicate.

Optimizers



Optimizers

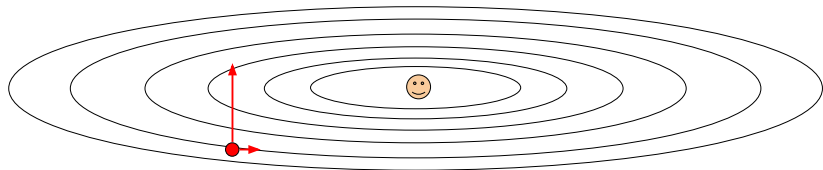
Suppose loss function is steep vertically but shallow horizontally:



Q: What is the trajectory along which we converge towards the minimum with SGD?

Optimizers

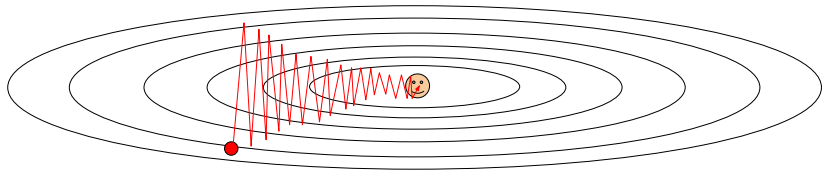
Suppose loss function is steep vertically but shallow horizontally:



Q: What is the trajectory along which we converge towards the minimum with SGD?

Optimizers

Suppose loss function is steep vertically but shallow horizontally:



Q: What is the trajectory along which we converge towards the minimum with SGD? **very slow progress along flat direction, jitter along steep one**

Optimizers

Momentum update

```
# Gradient descent update  
x += - learning_rate * dx
```



```
# Momentum update  
v = mu * v - learning_rate * dx # integrate velocity  
x += v # integrate position
```

- Physical interpretation as ball rolling down the loss function + friction (μ coefficient).
- μ = usually ~ 0.5 , 0.9 , or 0.99 (Sometimes annealed over time, e.g. from $0.5 \rightarrow 0.99$)

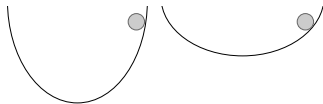
Optimizers

Momentum update

```
# Gradient descent update  
x += - learning_rate * dx
```

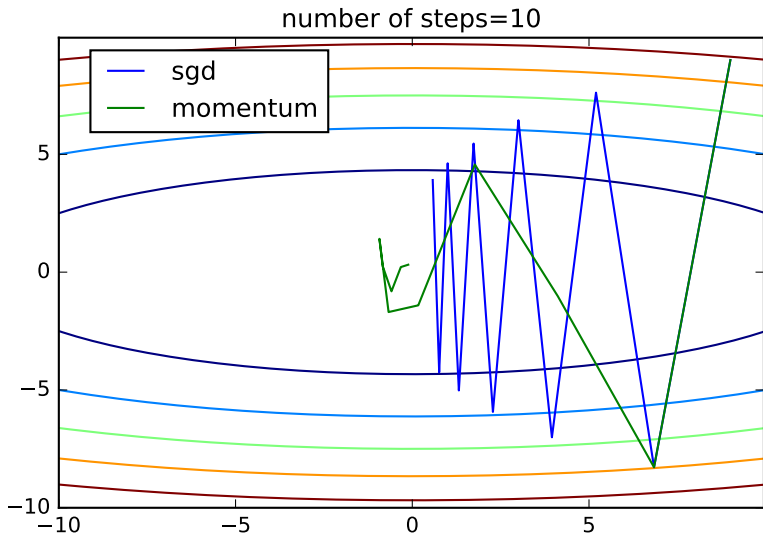


```
# Momentum update  
v = mu * v - learning_rate * dx # integrate velocity  
x += v # integrate position
```



- Allows a velocity to “build up” along shallow directions
- Velocity becomes damped in steep direction due to quickly changing sign

Optimizers

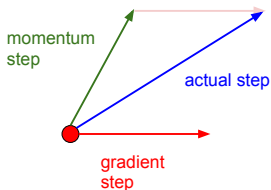


Optimizers

Nesterov Momentum update

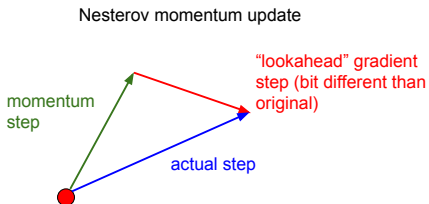
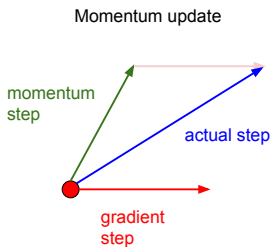
```
# Momentum update
v = mu * v - learning_rate * dx # integrate velocity
x += v # integrate position
```

Ordinary momentum update:



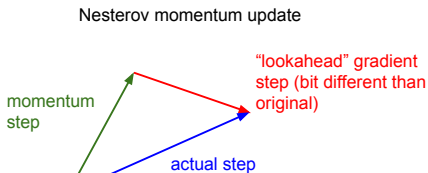
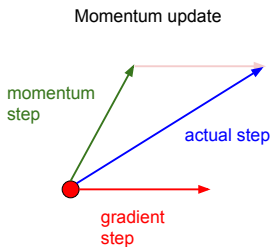
Optimizers

Nesterov Momentum update



Optimizers

Nesterov Momentum update



Nesterov: the only difference...

$$v_t = \mu v_{t-1} - \epsilon \nabla f(\theta_{t-1}) + \mu v_{t-1}$$

$$\theta_t = \theta_{t-1} + v_t$$

Optimizers

Nesterov Momentum update

$$v_t = \mu v_{t-1} - \epsilon \nabla f(\theta_{t-1} + \mu v_{t-1})$$

$$\theta_t = \theta_{t-1} + v_t$$

Slightly inconvenient...
usually we have :

$$\theta_{t-1}, \nabla f(\theta_{t-1})$$

Optimizers

Nesterov Momentum update

$$v_t = \mu v_{t-1} - \epsilon \nabla f(\theta_{t-1} + \mu v_{t-1})$$

$$\theta_t = \theta_{t-1} + v_t$$

Slightly inconvenient...
usually we have :

$$\theta_{t-1}, \nabla f(\theta_{t-1})$$

Variable transform and rearranging saves the day:

$$\phi_{t-1} = \theta_{t-1} + \mu v_{t-1}$$

Optimizers

Nesterov Momentum update

$$v_t = \mu v_{t-1} - \epsilon \nabla f(\theta_{t-1} + \mu v_{t-1})$$

$$\theta_t = \theta_{t-1} + v_t$$

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Variable transform and rearranging saves the day:

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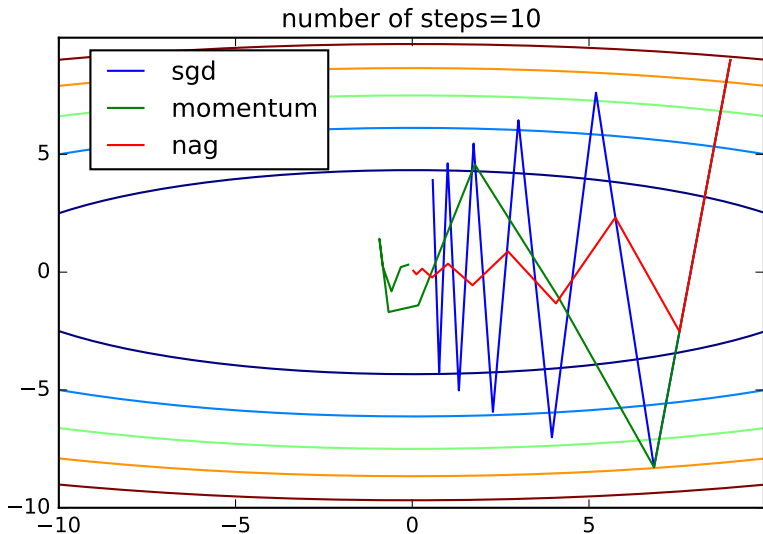
Replace all thetas with phis, rearrange and obtain:

$$v_t = \mu v_{t-1} - \epsilon \nabla f(\phi_{t-1})$$

$$\phi_t = \phi_{t-1} - \mu v_{t-1} + (1 + \mu)v_t$$

```
# Nesterov momentum update rewrite
v_prev = v
v = mu * v - learning_rate * dx
x += -mu * v_prev + (1 + mu) * v
```

Optimizers



Optimizers

AdaGrad update

[Duchi et al., 2011]

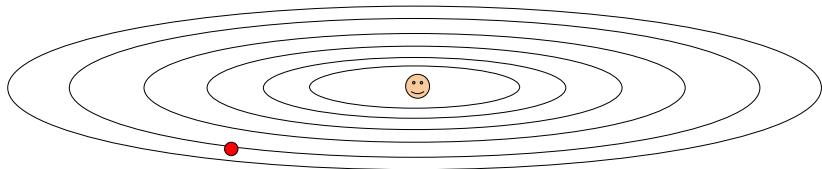
```
# Adagrad update
cache += dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

Added element-wise scaling of the gradient based on the historical sum of squares in each dimension

Optimizers

AdaGrad update

```
# Adagrad update  
cache += dx**2  
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

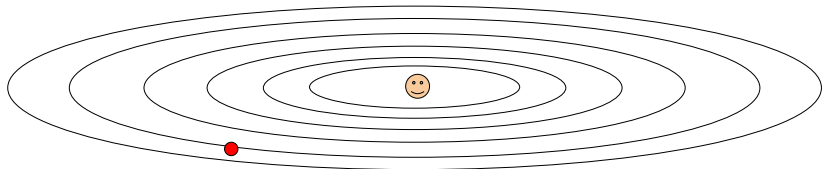


Q: What happens with AdaGrad?

Optimizers

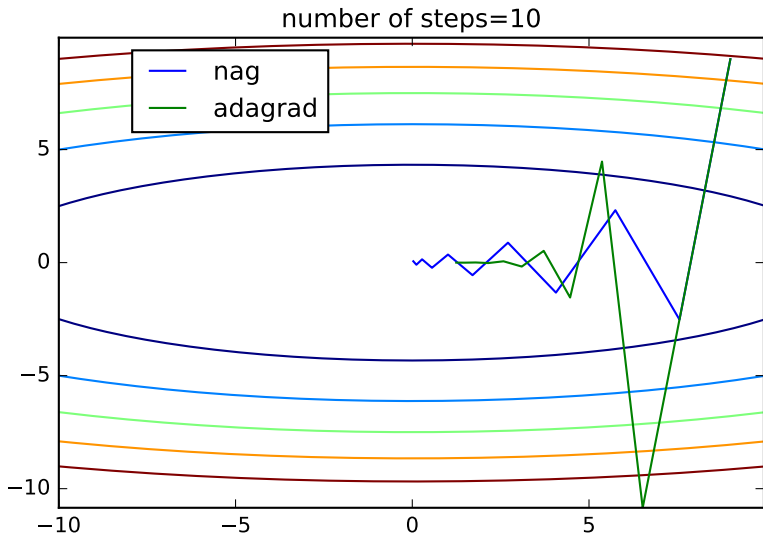
AdaGrad update

```
# Adagrad update  
cache += dx**2  
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```



Q2: What happens to the step size over long time?

Optimizers



Optimizers

RMSProp update

[Tieleman and Hinton, 2012]

```
# Adagrad update  
cache += dx**2  
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```



```
# RMSProp  
cache = decay_rate * cache + (1 - decay_rate) * dx**2  
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```


Optimizers

rmsprop: A mini-batch version of rprop

- rprop is equivalent to using the gradient but also dividing by the size of the gradient.
 - The problem with mini-batch rprop is that we divide by a different number for each mini-batch. So why not force the number we divide by to be very similar for adjacent mini-batches?
- rmsprop: Keep a moving average of the squared gradient for each weight
$$MeanSquare(w, t) = 0.9 MeanSquare(w, t-1) + 0.1 \left(\frac{\partial E}{\partial w}(t) \right)^2$$
- Dividing the gradient by $\sqrt{MeanSquare(w, t)}$ makes the learning work much better (Tijmen Tieleman, unpublished).

Introduced in a slide in Geoff Hinton's Coursera class, lecture 6

Optimizers

rmsprop: A mini-batch version of rprop

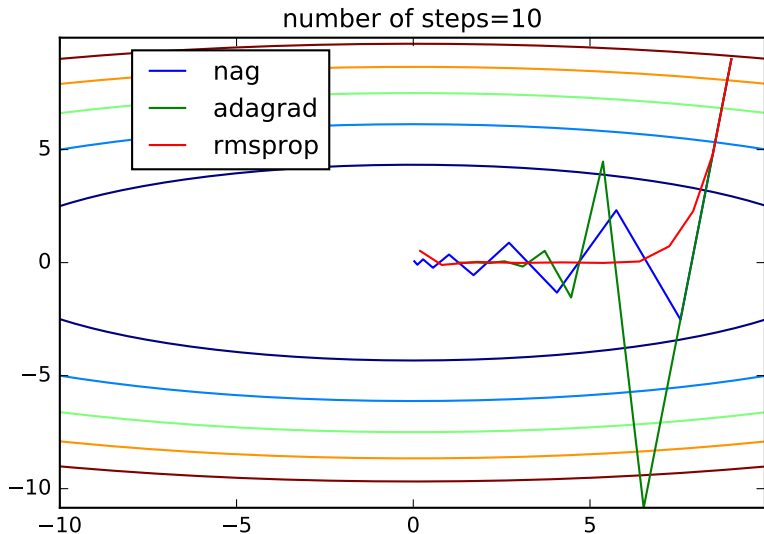
- rprop is equivalent to using the gradient but also dividing by the size of the gradient.
 - The problem with mini-batch rprop is that we divide by a different number for each mini-batch. So why not force the number we divide by to be very similar for adjacent mini-batches?
- rmsprop: Keep a moving average of the squared gradient for each weight
$$MeanSquare(w, t) = 0.9 MeanSquare(w, t-1) + 0.1 \left(\frac{\partial E}{\partial w}(t) \right)^2$$
- Dividing the gradient by $\sqrt{MeanSquare(w, t)}$ makes the learning work much better (Tijmen Tieleman, unpublished).

Introduced in a slide in Geoff Hinton's Coursera class, lecture 6

Cited by several papers as:

[52] T. Tieleman and G. E. Hinton. Lecture 6.5-rmsprop: Divide the gradient by a running average of its recent magnitude., 2012.

Optimizers



Optimizers

Adam update

[Kingma and Ba, 2014]

(incomplete, but close)

```
# Adam
m = beta1*m + (1-beta1)*dx # update first moment
v = beta2*v + (1-beta2)*(dx**2) # update second moment
x += - learning_rate * m / (np.sqrt(v) + 1e-7)
```

Optimizers

Adam update

[Kingma and Ba, 2014]

(incomplete, but close)

```
# Adam
m = beta1*m + (1-beta1)*dx # update first moment
v = beta2*v + (1-beta2)*(dx**2) # update second moment
x += - learning_rate * m / (np.sqrt(v) + 1e-7)
```

momentum

RMSProp-like

Looks a bit like RMSProp with momentum

Optimizers

Adam update

[Kingma and Ba, 2014]

(incomplete, but close)

```
# Adam
m = beta1*m + (1-beta1)*dx # update first moment
v = beta2*v + (1-beta2)*(dx**2) # update second moment
x += - learning_rate * m / (np.sqrt(v) + 1e-7)
```

momentum

RMSProp-like

Looks a bit like RMSProp with momentum

```
# RMSProp
cache = decay_rate * cache + (1 - decay_rate) * dx**2
x += - learning_rate * dx / (np.sqrt(cache) + 1e-7)
```

Optimizers

Adam update

[Kingma and Ba, 2014]

```
# Adam
m,v = #... initialize caches to zeros
for t in xrange(1, big_number):
    dx = # ... evaluate gradient
    m = beta1*m + (1-beta1)*dx # update first moment
    v = beta2*v + (1-beta2)*(dx**2) # update second moment
    mb = m/(1-beta1**t) # correct bias
    vb = v/(1-beta2**t) # correct bias
    x += - learning_rate * mb / (np.sqrt(vb) + 1e-7)
```

momentum

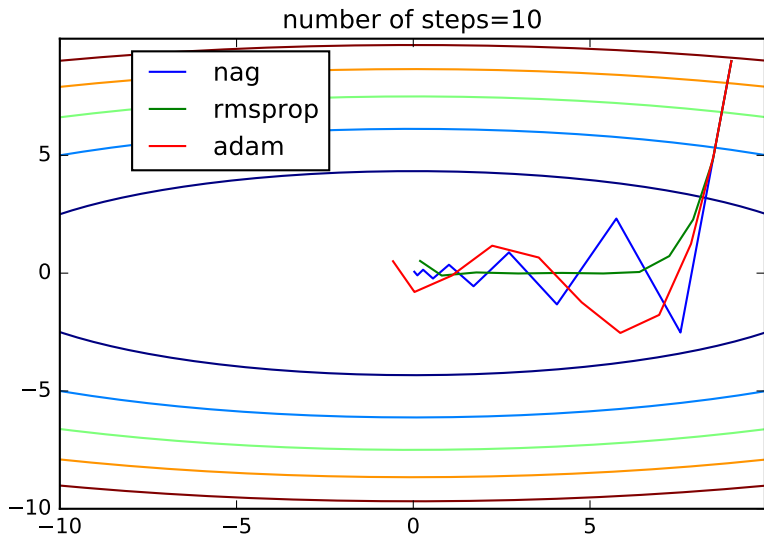
bias correction

(only relevant in first few iterations when t is small)

RMSProp-like

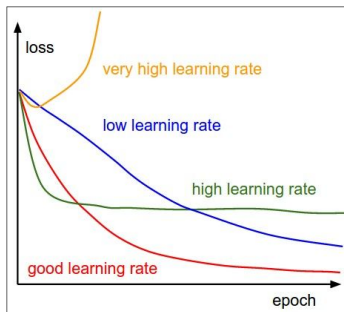
The bias correction compensates for the fact that m, v are initialized at zero and need some time to “warm up”.

Optimizers



Optimizers

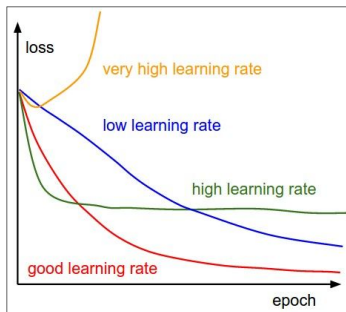
SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



Q: Which one of these learning rates is best to use?

Optimizers

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



=> **Learning rate decay over time!**

step decay:

e.g. decay learning rate by half every few epochs.

exponential decay:

$$\alpha = \alpha_0 e^{-kt}$$

1/t decay:

$$\alpha = \alpha_0 / (1 + kt)$$

Optimizers

Second order optimization methods

second-order Taylor expansion:

$$J(\boldsymbol{\theta}) \approx J(\boldsymbol{\theta}_0) + (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^\top \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}_0) + \frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^\top \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

Solving for the critical point we obtain the Newton parameter update:

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}_0 - \mathbf{H}^{-1} \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}_0)$$

Q: what is nice about this update?

Optimizers

Second order optimization methods

second-order Taylor expansion:

$$J(\boldsymbol{\theta}) \approx J(\boldsymbol{\theta}_0) + (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^\top \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}_0) + \frac{1}{2} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)^\top \mathbf{H} (\boldsymbol{\theta} - \boldsymbol{\theta}_0)$$

Solving for the critical point we obtain the Newton parameter update:

$$\boldsymbol{\theta}^* = \boldsymbol{\theta}_0 - \mathbf{H}^{-1} \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}_0)$$

notice:

no hyperparameters! (e.g. learning rate)

Q2: why is this impractical for training Deep Neural Nets?

Optimizers

Second order optimization methods

$$\theta^* = \theta_0 - H^{-1} \nabla_{\theta} J(\theta_0)$$

- Quasi-Newton methods (**BGFS** most popular):
instead of inverting the Hessian ($O(n^3)$), approximate inverse Hessian with rank 1 updates over time ($O(n^2)$ each).
- **L-BFGS** (Limited memory BFGS):
Does not form/store the full inverse Hessian.

Optimizers

L-BFGS

- **Usually works very well in full batch, deterministic mode** i.e. if you have a single, deterministic $f(x)$ then L-BFGS will probably work very nicely
- **Does not transfer very well to mini-batch setting.** Gives bad results. Adapting L-BFGS to large-scale, stochastic setting is an active area of research.

Optimizers

In practice:

- **Adam** is a good default choice in most cases
- If you can afford to do full batch updates then try out **L-BFGS** (and don't forget to disable all sources of noise)

Demo

Jupyter notebook demo

Debugging optimizer

Double check that the loss is reasonable:

```
def init_two_layer_model(input_size, hidden_size, output_size):
    # initialize a model
    model = {}
    model['W1'] = 0.0001 * np.random.randn(input_size, hidden_size)
    model['b1'] = np.zeros(hidden_size)
    model['W2'] = 0.0001 * np.random.randn(hidden_size, output_size)
    model['b2'] = np.zeros(output_size)
    return model
```

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
loss, grad = two_layer_net(X_train, model, y_train, 1e3) # crank up regularization
print loss
```

3.06859716482

loss went up, good. (sanity check)

Debugging optimizer

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the training data

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
                                  model, two_layer_net,
                                  num_epochs=200, reg=0.0,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = False,
                                  learning_rate=1e-3, verbose=True)
```

The above code:

- take the first 20 examples from CIFAR-10
- turn off regularization (reg = 0.0)
- use simple vanilla 'sgd'

Debugging optimizer

Lets try to train now...

Tip: Make sure that you can overfit very small portion of the training data

Very small loss, train accuracy 1.00, nice!

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
X_tiny = X_train[:20] # take 20 examples
y_tiny = y_train[:20]
best_model, stats = trainer.train(X_tiny, y_tiny, X_tiny, y_tiny,
    model, two_layer_net,
    num_epochs=200, reg=0.0,
    update='sgd', learning_rate_decay=1,
    sample_batches = False,
    learning_rate=1e-3, verbose=True)
```

```
Finished epoch 1 / 200: cost 2.302603, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 2 / 200: cost 2.302258, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 3 / 200: cost 2.301849, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 4 / 200: cost 2.301196, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 5 / 200: cost 2.300044, train: 0.650000, val 0.650000, lr 1.000000e-03
Finished epoch 6 / 200: cost 2.297864, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 7 / 200: cost 2.293595, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 8 / 200: cost 2.285096, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 9 / 200: cost 2.268094, train: 0.550000, val 0.550000, lr 1.000000e-03
Finished epoch 10 / 200: cost 2.234787, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 11 / 200: cost 2.173187, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 12 / 200: cost 2.076862, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 13 / 200: cost 1.974090, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 14 / 200: cost 1.895885, train: 0.400000, val 0.400000, lr 1.000000e-03
Finished epoch 15 / 200: cost 1.820876, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 16 / 200: cost 1.737430, train: 0.450000, val 0.450000, lr 1.000000e-03
Finished epoch 17 / 200: cost 1.642356, train: 0.500000, val 0.500000, lr 1.000000e-03
Finished epoch 18 / 200: cost 1.535239, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 19 / 200: cost 1.421527, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 20 / 200: cost 1.307815, train: 0.600000, val 0.600000, lr 1.000000e-03
Finished epoch 195 / 200: cost 0.002694, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 196 / 200: cost 0.002674, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 197 / 200: cost 0.002655, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 198 / 200: cost 0.002635, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 199 / 200: cost 0.002617, train: 1.000000, val 1.000000, lr 1.000000e-03
Finished epoch 200 / 200: cost 0.002597, train: 1.000000, val 1.000000, lr 1.000000e-03
finished optimization. best validation accuracy: 1.000000
```

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e-6, verbose=True)
```

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

```

model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best validation accuracy: 0.192000

```

Loss barely changing

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```

model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best_validation accuracy: 0.192000

```

Loss barely changing: Learning rate is probably too low

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```

model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches=True,
                                  learning_rate=1e-6, verbose=True)
Finished epoch 1 / 10: cost 2.302576, train: 0.080000, val 0.103000, lr 1.000000e-06
Finished epoch 2 / 10: cost 2.302582, train: 0.121000, val 0.124000, lr 1.000000e-06
Finished epoch 3 / 10: cost 2.302558, train: 0.119000, val 0.138000, lr 1.000000e-06
Finished epoch 4 / 10: cost 2.302519, train: 0.127000, val 0.151000, lr 1.000000e-06
Finished epoch 5 / 10: cost 2.302517, train: 0.158000, val 0.171000, lr 1.000000e-06
Finished epoch 6 / 10: cost 2.302518, train: 0.179000, val 0.172000, lr 1.000000e-06
Finished epoch 7 / 10: cost 2.302466, train: 0.180000, val 0.176000, lr 1.000000e-06
Finished epoch 8 / 10: cost 2.302452, train: 0.175000, val 0.185000, lr 1.000000e-06
Finished epoch 9 / 10: cost 2.302459, train: 0.206000, val 0.192000, lr 1.000000e-06
Finished epoch 10 / 10: cost 2.302420, train: 0.190000, val 0.192000, lr 1.000000e-06
finished optimization. best_validation accuracy: 0.192000
  
```

Loss barely changing: Learning rate is probably too low

Notice train/val accuracy goes to 20% though, what's up with that? (remember this is softmax)

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low

```
model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=1e6, verbose=True)
```

Okay now lets try learning rate 1e6. What could possibly go wrong?

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```

model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                model, two_layer_net,
                                num_epochs=10, reg=0.000001,
                                update='sgd', learning_rate_decay=1,
                                sample_batches = True,
                                learning_rate=1e6, verbose=True)
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:50: RuntimeWarning: divide by zero en
countered in log
  data_loss = -np.sum(np.log(probs[range(N), y])) / N
/home/karpathy/cs231n/code/cs231n/classifiers/neural_net.py:48: RuntimeWarning: invalid value enc
ountered in subtract
  probs = np.exp(scores - np.max(scores, axis=1, keepdims=True))
Finished epoch 1 / 10: cost nan, train: 0.091000, val 0.087000, lr 1.000000e+06
Finished epoch 2 / 10: cost nan, train: 0.095000, val 0.087000, lr 1.000000e+06
Finished epoch 3 / 10: cost nan, train: 0.100000, val 0.087000, lr 1.000000e+06

```

cost: NaN almost
always means high
learning rate...

Debugging optimizer

Lets try to train now...

I like to start with small regularization and find learning rate that makes the loss go down.

loss not going down:
learning rate too low
loss exploding:
learning rate too high

```

model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
trainer = ClassifierTrainer()
best_model, stats = trainer.train(X_train, y_train, X_val, y_val,
                                  model, two_layer_net,
                                  num_epochs=10, reg=0.000001,
                                  update='sgd', learning_rate_decay=1,
                                  sample_batches = True,
                                  learning_rate=3e-3, verbose=True)
Finished epoch 1 / 10: cost 2.186654, train: 0.308000, val 0.306000, lr 3.000000e-03
Finished epoch 2 / 10: cost 2.176230, train: 0.330000, val 0.350000, lr 3.000000e-03
Finished epoch 3 / 10: cost 1.942257, train: 0.376000, val 0.352000, lr 3.000000e-03
Finished epoch 4 / 10: cost 1.827868, train: 0.329000, val 0.310000, lr 3.000000e-03
Finished epoch 5 / 10: cost inf, train: 0.128000, val 0.128000, lr 3.000000e-03
Finished epoch 6 / 10: cost inf, train: 0.144000, val 0.147000, lr 3.000000e-03

```

3e-3 is still too high. Cost explodes....

=> Rough range for learning rate we should be cross-validating is somewhere [1e-3 ... 1e-5]

Hyperparameter optimization

Cross-validation strategy

I like to do **coarse** -> **fine** cross-validation in stages

First stage: only a few epochs to get rough idea of what params work

Second stage: longer running time, finer search

... (repeat as necessary)

Tip for detecting explosions in the solver:

If the cost is ever $> 3 * \text{original cost}$, break out early

Hyperparameter optimization

For example: run coarse search for 5 epochs

```

max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)

    trainer = ClassifierTrainer()
    model = init_two_layer_model(32*32*3, 50, 10) # input size, hidden size, number of classes
    trainer = ClassifierTrainer()
    best_model_local, stats = trainer.train(X_train, y_train, X_val, y_val,
                                           model, two_layer_net,
                                           num_epochs=5, reg=reg,
                                           update='momentum', learning_rate_decay=0.9,
                                           sample_batches = True, batch_size = 100,
                                           learning_rate=lr, verbose=False)
  
```

note it's best to optimize
in log space!

```

val acc: 0.412000, lr: 1.405206e-04, reg: 4.793564e-01, (1 / 100)
val acc: 0.214000, lr: 7.231888e-06, reg: 2.321281e-04, (2 / 100)
val acc: 0.208000, lr: 2.119571e-06, reg: 8.011857e+01, (3 / 100)
val acc: 0.196000, lr: 1.551131e-05, reg: 4.374936e-05, (4 / 100)
val acc: 0.079000, lr: 1.753300e-05, reg: 1.200424e+03, (5 / 100)
val acc: 0.223000, lr: 4.215128e-05, reg: 4.196174e+01, (6 / 100)
val acc: 0.441000, lr: 1.750259e-04, reg: 2.110807e-04, (7 / 100)
val acc: 0.241000, lr: 6.749231e-05, reg: 4.226413e+01, (8 / 100)
val acc: 0.482000, lr: 4.296863e-04, reg: 6.642555e-01, (9 / 100)
val acc: 0.079000, lr: 5.401602e-06, reg: 1.599828e+04, (10 / 100)
val acc: 0.154000, lr: 1.618508e-06, reg: 4.925252e-01, (11 / 100)
  
```

nice

Hyperparameter optimization

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)
```

adjust range



```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-4, 0)
    lr = 10**uniform(-3, -4)
```

```
val_acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
val_acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
val_acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
val_acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
val_acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
val_acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
val_acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
val_acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
val_acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)
```

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

Hyperparameter optimization

Now run finer search...

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-5, 5)
    lr = 10**uniform(-3, -6)
```

adjust range

```
max_count = 100
for count in xrange(max_count):
    reg = 10**uniform(-4, 0)
    lr = 10**uniform(-3, -4)
```

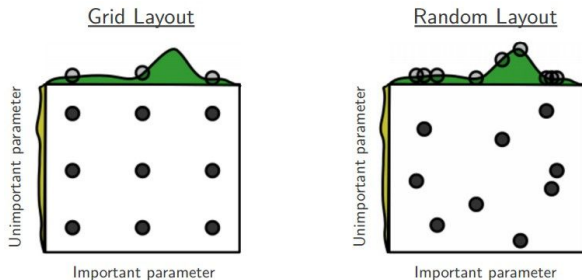
```
val_acc: 0.527000, lr: 5.340517e-04, reg: 4.097824e-01, (0 / 100)
val_acc: 0.492000, lr: 2.279484e-04, reg: 9.991345e-04, (1 / 100)
val_acc: 0.512000, lr: 8.680827e-04, reg: 1.349727e-02, (2 / 100)
val_acc: 0.461000, lr: 1.028377e-04, reg: 1.220193e-02, (3 / 100)
val_acc: 0.460000, lr: 1.113730e-04, reg: 5.244309e-02, (4 / 100)
val_acc: 0.498000, lr: 9.477776e-04, reg: 2.001293e-03, (5 / 100)
val_acc: 0.469000, lr: 1.484369e-04, reg: 4.328313e-01, (6 / 100)
val_acc: 0.522000, lr: 5.586261e-04, reg: 2.312685e-04, (7 / 100)
val_acc: 0.530000, lr: 5.808183e-04, reg: 8.259964e-02, (8 / 100)
val_acc: 0.489000, lr: 1.979168e-04, reg: 1.010889e-04, (9 / 100)
val_acc: 0.490000, lr: 2.036031e-04, reg: 2.406271e-03, (10 / 100)
val_acc: 0.475000, lr: 2.021162e-04, reg: 2.287807e-01, (11 / 100)
val_acc: 0.460000, lr: 1.135527e-04, reg: 3.905040e-02, (12 / 100)
val_acc: 0.515000, lr: 6.947668e-04, reg: 1.562808e-02, (13 / 100)
val_acc: 0.531000, lr: 9.471549e-04, reg: 1.433895e-03, (14 / 100)
val_acc: 0.509000, lr: 3.140888e-04, reg: 2.857518e-01, (15 / 100)
val_acc: 0.514000, lr: 6.438349e-04, reg: 3.033781e-01, (16 / 100)
val_acc: 0.502000, lr: 3.921784e-04, reg: 2.707126e-04, (17 / 100)
val_acc: 0.509000, lr: 9.752279e-04, reg: 2.850865e-03, (18 / 100)
val_acc: 0.500000, lr: 2.412048e-04, reg: 4.997821e-04, (19 / 100)
val_acc: 0.466000, lr: 1.319314e-04, reg: 1.189915e-02, (20 / 100)
val_acc: 0.516000, lr: 8.039527e-04, reg: 1.528291e-02, (21 / 100)
```

53% - relatively good
for a 2-layer neural net
with 50 hidden neurons.

But this best cross-
validation result is
worrying. Why?

Hyperparameter optimization

Random Search vs. Grid Search



Random Search for Hyper-Parameter Optimization
Bergstra and Bengio, 2012

Conclusions (What we know in 2017)

- BP is just chain rule in calculus
- Use ReLU. Never use Sigmoid (use Tanh instead)
- Input preprocessing is no longer very important
 - Do subtract mean
 - Whitening and normalizing are not much needed
- Weight initialization on the other hand is extremely important for deep networks
- Use batch normalization if you can
- Use dropout
- Use Adam (or maybe RMSprop) for optimizer. If you don't have much data, can consider LBFGS
- Need to babysit your learning for real-world problems
- Never use grid search for tuning your hyperparameters

Logistics

Give me your presentation preference and I'll throw the “dice” now