Recurrent Neural Networks

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(Slides credit to Stanford CS231n and Hinton et al.)

Table of Contents

Review and Overview

- We looked into couple use cases of CNNs previously
  - Recognition and localization
  - Object detection
  - Some use of CNNs for arts
- Up to now, the network models we have studied are all memoryless
  - We will discuss a non-memoryless model—recurrent neural networks today

Why non-memoryless models

- Almost all natural signals are sequential if we take time into account (we just cannot escape time)
  - Memory is needed to remember the past
- They also offer a simplified solution for some problems (for example, number addition)
- They can treat some unsupervised problems as supervised problems
  - Consider prediction of a stock: unsupervised? Supervised?

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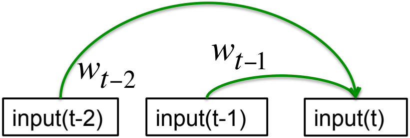
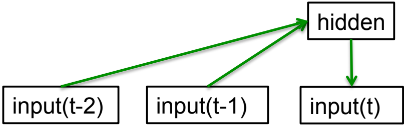
Engineering hacks [Hinton 2012, week 7]

Memoryless models for sequences

- Autoregressive models
  Predict the next term in a sequence from a fixed number of previous terms using "delay taps".



- Feed-forward neural nets
  These generalize autoregressive models by using one or more layers of non-linear hidden units. *e.g.* Bengio's first language model.



Non-memoryless models

- Benefit: memories increase the expressive power of the model
- Typically we do not know the exact values of the hidden states (that is why “hidden”). In many cases, the best we could do is just to infer a probability distribution over the hidden states
- Let’s look at two classic examples

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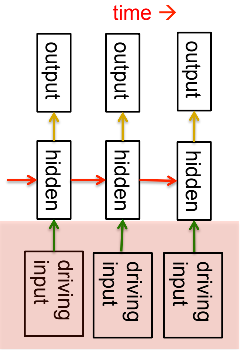
Linear dynamical systems (Engineers love them!)



- These are generative models with real continuous values as hidden states that cannot be observed directly
  - The hidden state has linear dynamics with Gaussian noise and produces the observations subjected to linear Gaussian noise
  - There can also be driving inputs
- To predict next output, we need to infer the hidden state

[Hinton 2012, Week 7]

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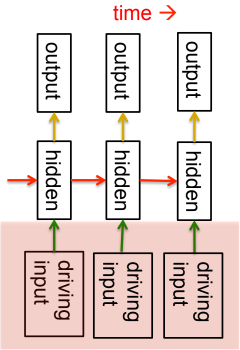

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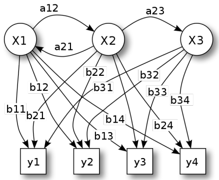
State-Space Models (Computer scientists love them!)



- State-Space Models or Hidden Markov Models (HMMs) have a discrete one-of-N hidden state. Transitions between states are stochastic and controlled by a transition matrix. The output produced by a state are also stochastic
  - We don't know which state produced a given output. So the state is "hidden"
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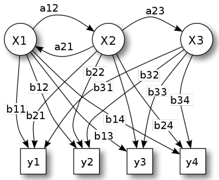

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A fundamental limitation of state space models

- The only information stored in the model is which state the model currently is in
  - So with N hidden states it can only remember a maximum $\log(N)$ bits of information
- Consider the speech prediction of one half from earlier half
  - The syntax needs to fit (e.g. number and tense agreement)
  - The semantics needs to fit. The intonation needs to fit
  - The accent, rate, volume, and vocal tract characteristics must all fit
- All these aspects combined could be 100 bits of information that the first half of an utterance needs to convey to the second half 2^{100} states

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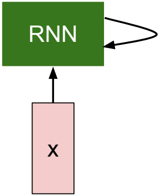
Recurrent neural networks (RNNs)

Recurrent Neural Network



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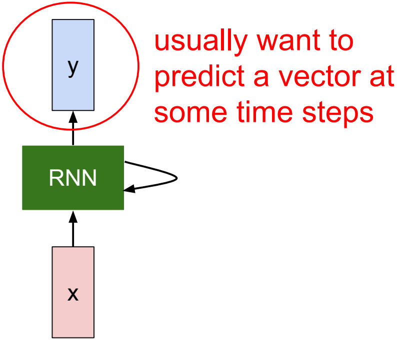

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Recurrent neural networks (RNNs)

Recurrent Neural Network

We can process a sequence of vectors **x** by applying a recurrence formula at every time step:

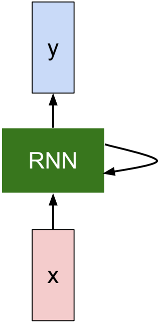
$$\boxed{h_t} = \boxed{f_W}(\boxed{h_{t-1}}, \boxed{x_t})$$

new state

old state

input vector at some time step

some function with parameters W



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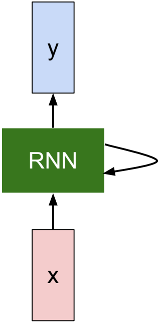

Notice: the same function and the same set of parameters are used at every time step.

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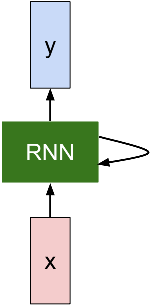
Recurrent neural networks (RNNs)

(Vanilla) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:



$$h_t = f_W(h_{t-1}, x_t)$$

$$h_t = \tanh(W_{hh} h_{t-1} + W_{xh} x_t)$$

$$y_t = W_{hy} h_t$$

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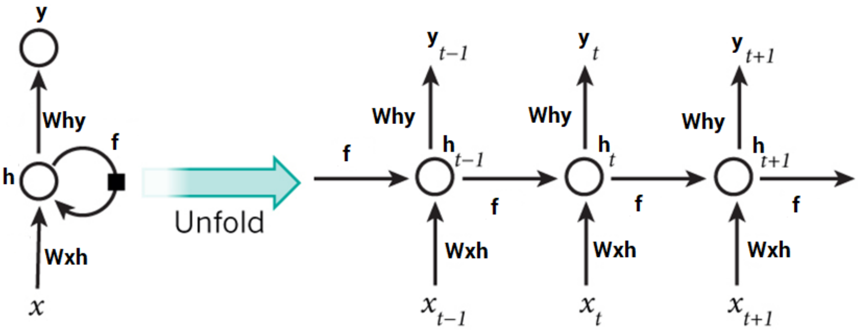
Back-Propagation Through Time (BPTT)

- For training, we can unroll all the time step to form a stack of activities and backprop will then similar to regular backprop
- The backward pass peels activities off the stack to compute the error derivatives at each time step
- After the backward pass we add together the derivatives at all the different times for each weight



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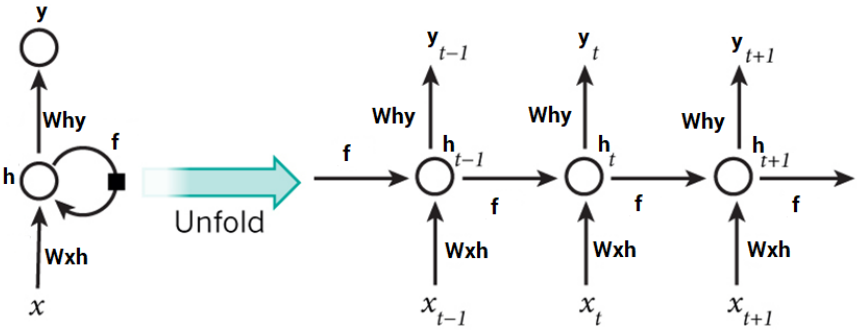

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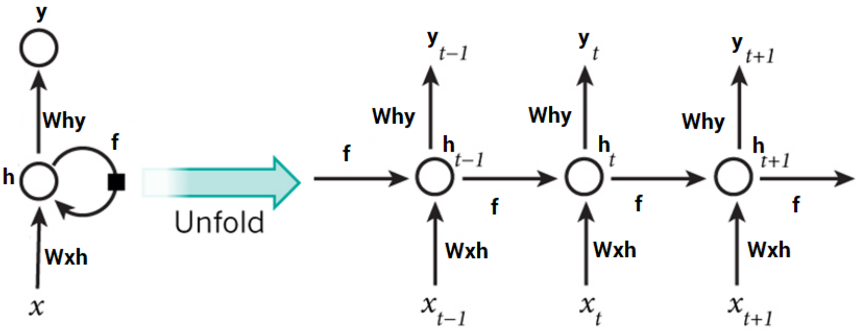

An irritative extra issue

- We need to specify the initial activity state of all the hidden and output units
- We could just fix these initial states to have some default value like 0.5
- But it is better to treat the initial states as learned parameters
- We learn them in the same way as we learn the weights
  - Start off with an initial random guess for the initial states
  - At the end of each training sequence, backpropagate through time all the way to the initial states to get the gradient of the error function with respect to each initial state
  - Adjust the initial states by following the negative gradient

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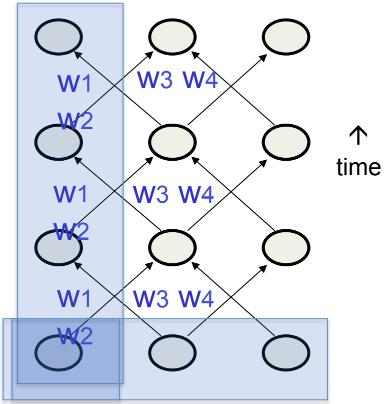
Providing inputs to recurrent networks



- We can specify inputs in several ways:
  - Specify the initial states of all the units
  - Specify the initial states of a subset of the units
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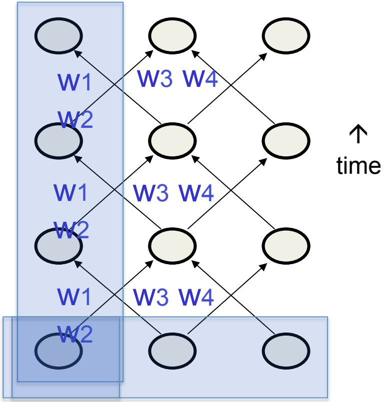

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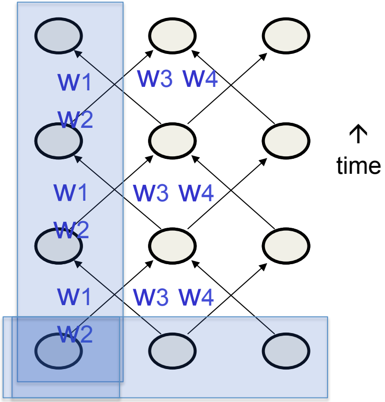

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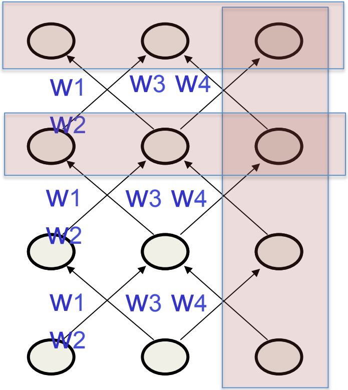
Teaching recurrent networks to learn signals



- We can specify targets in several ways:
  - Specify desired final activities of all the units
  - Specify desired activities of all units for the last few steps
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    - The other units are input or hidden units.

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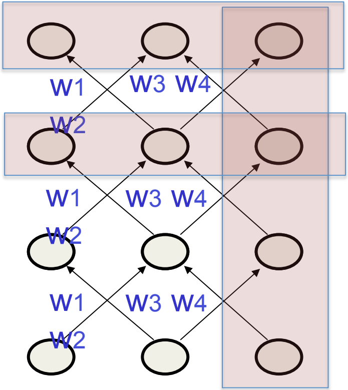

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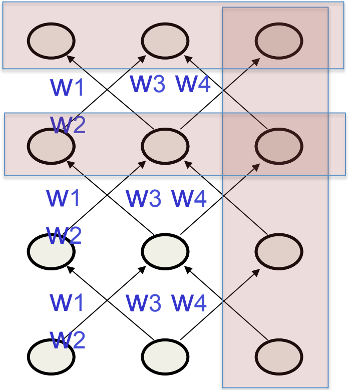

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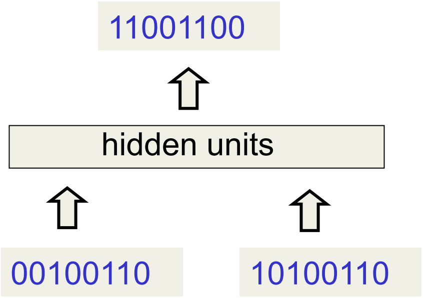
Toy problem for RNN: binary addition



- We can train a feedforward net to do binary addition, but...
  - We must decide in advance the maximum number of digits in each number
  - We expect weights to process different bits to be the same, but it is tricky to enforce that
- As a result, feedforward nets do not generalize well for the binary addition task

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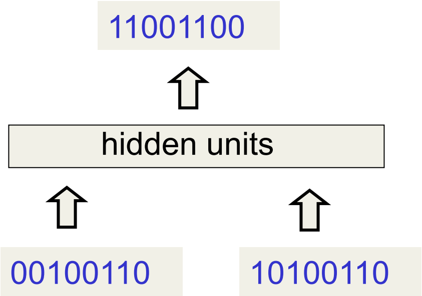

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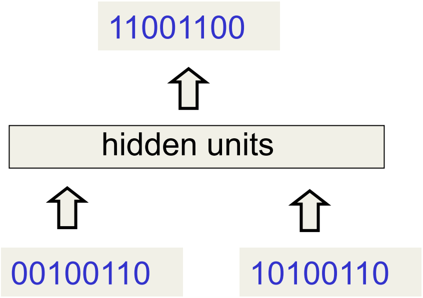

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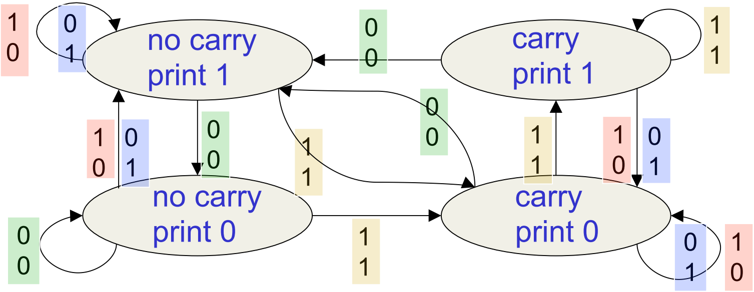
We are trying to learn this!

The algorithm for binary addition



This is a finite state automaton. It decides what transition to make by looking at the next column. It prints after making the transition. It moves from right to left over the two input numbers.

A little bit detail

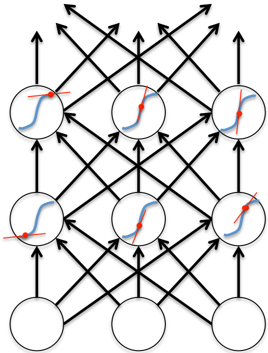
$$x = [b_8, b_7, \cdots, b_1]$$
$$y = [c_8, c_7, \cdots, c_1]$$
$$z = x + y = [d_8, d_7, \cdots, d_1]$$
$$\hat{z} = [\hat{d}_8, \hat{d}_7, \cdots, \hat{d}_1]$$

Hidden unit: $h_i = sigm(W_{x,h}[b_i, c_i]^T + W_{h,h}h_{i-1})$

Output: $\hat{d}_i = sigm(W_{h,z}h_i)$

https://github.com/llSourcell/recurrent_neural_net_demo

Why training RNN is difficulty? The backward pass is linear



- There is a big difference between the forward and backward passes
- In the forward pass we use squashing functions (like the logistic) to prevent the activity vectors from exploding
- The backward pass, is completely linear. If you double the error derivatives at the final layer, all the error derivatives will double
  - The forward pass determines the slope of the linear function used for backpropagating through each neuron

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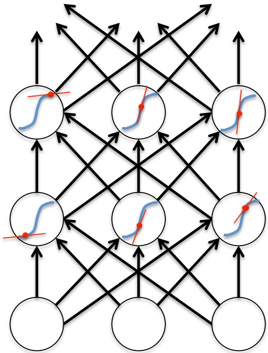

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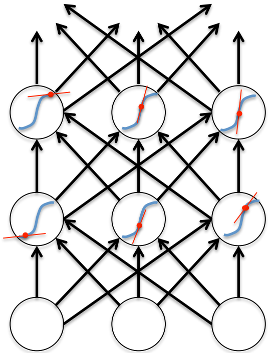

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The problem of exploding or vanishing gradients

- What happens to the magnitude of the gradients as we backpropagate through many layers?
  - If the weights are small, the gradients shrink exponentially.
  - If the weights are big the gradients grow exponentially
- Typical feed-forward neural nets can cope with these exponential effects when they only have a few hidden layers
- In an RNN trained on long sequences (e.g. 100 time steps) the gradients can easily explode or vanish
  - We could avoid this by initializing the weights very carefully
- Even with good initial weights, the dependency of the current target output from an input many time-steps ago tends to be numerically unstable
  - So RNNs have difficulty dealing with long-range dependencies

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Understanding gradient flow dynamics

Cute backprop signal video: http://imgur.com/gallery/vaNahKE

```
H = 5    # dimensionality of hidden state
T = 50   # number of time steps
Whh = np.random.randn(H,H)

# forward pass of an RNN (ignoring inputs x)
hs = {}
ss = {}
hs[-1] = np.random.randn(H)
for t in xrange(T):
    ss[t] = np.dot(Whh, hs[t-1])
    hs[t] = np.maximum(0, ss[t])

# backward pass of the RNN
dhs = {}
dss = {}
dhs[T-1] = np.random.randn(H) # start off the chain with random gradient
for t in reversed(xrange(T)):
    dss[t] = (hs[t] > 0) * dhs[t] # backprop through the nonlinearity
    dhs[t-1] = np.dot(Whh.T, dss[t]) # backprop into previous hidden state
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if the largest eigenvalue is > 1, gradient will explode
if the largest eigenvalue is < 1, gradient will vanish

[On the difficulty of training Recurrent Neural Networks, Pascanu et al., 2013]

Four effective ways to learn an RNN

- Long Short Term Memory: Make the RNN out of little modules that are designed to remember values for a long time
- Hessian Free Optimization: Deal with the vanishing gradients problem by using a fancy optimizer that can detect directions with a tiny gradient but even smaller curvature
  - The HF optimizer (Martens & Sutskever, 2011) is good at this
- Echo State Networks: Initialize the input→ hidden and hidden→hidden and output→ hidden connections very carefully so that the hidden state has a huge reservoir of weakly coupled oscillators which can be selectively driven by the input
  - ESNs only need to learn the hidden→output connections
- Good initialization with momentum: Initialize like in Echo State Networks, but then learn all of the connections using momentum

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- Hessian Free Optimization: Deal with the vanishing gradients problem by using a fancy optimizer that can detect directions with a tiny gradient but even smaller curvature
  - The HF optimizer (Martens & Sutskever, 2011) is good at this
- Echo State Networks: Initialize the input→ hidden and hidden→hidden and output→ hidden connections very carefully so that the hidden state has a huge reservoir of weakly coupled oscillators which can be selectively driven by the input
  - ESNs only need to learn the hidden→output connections
- Good initialization with momentum: Initialize like in Echo State Networks, but then learn all of the connections using momentum

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Long Short Term Memory (LSTM)

- Hochreiter & Schmidhuber (1997) solved the problem of getting an RNN to remember things for a long time (like hundreds of time steps)
  - Keep short-term memory for a long period of time, thus the name
- They designed a memory cell using logistic and linear units with multiplicative interactions

- Information gets into the cell whenever its "write" gate is on
- The information stays in the cell so long as its "keep" gate is on
- Information can be read from the cell by turning on its "read" gate

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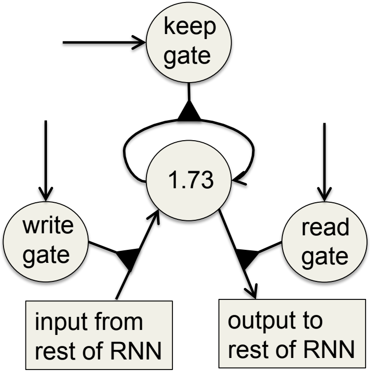
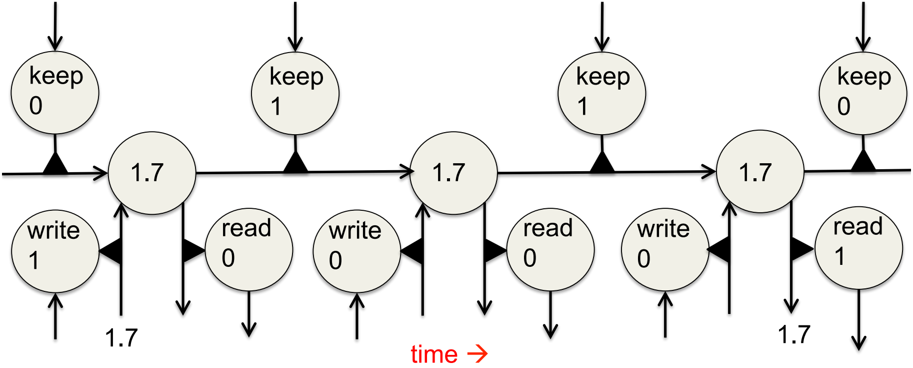
Implementing a memory cell in a neural network



- To preserve information for a long time in the activities of an RNN, we use a circuit mimicking an analog memory cell
  - Information is kept in the cell when "keep" gate is on
  - Information is stored in the cell by activating its write gate
  - Information is retrieved by activating the read gate
  - We can backpropagate through this circuit because logistics are have nice derivatives

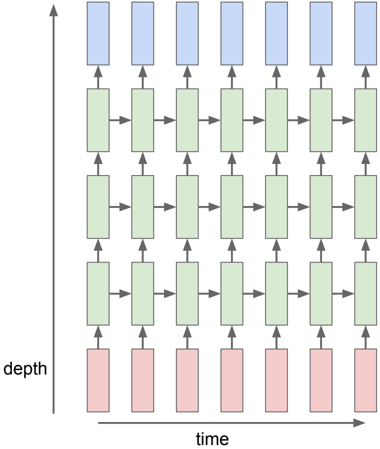
Backpropagation through a memory cell



RNN:

$$\boxed{h_t^l = \tanh W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}}$$

$h \in \mathbb{R}^n$. W^l $[n \times 2n]$



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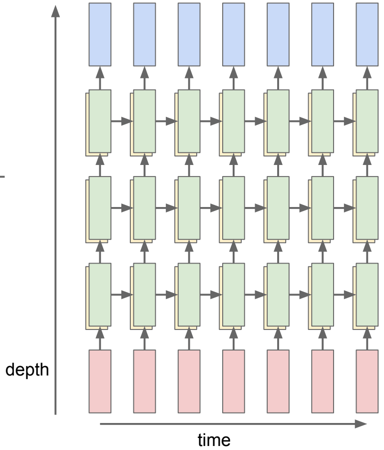
LSTM:

$W^l \ [4n \times 2n]$

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \text{sigm} \\ \text{sigm} \\ \text{sigm} \\ \tanh \end{pmatrix} W^l \begin{pmatrix} h_t^{l-1} \\ h_{t-1}^l \end{pmatrix}$$

$$c_t^l = f \odot c_{t-1}^l + i \odot g$$

$$h_t^l = o \odot \tanh(c_t^l)$$



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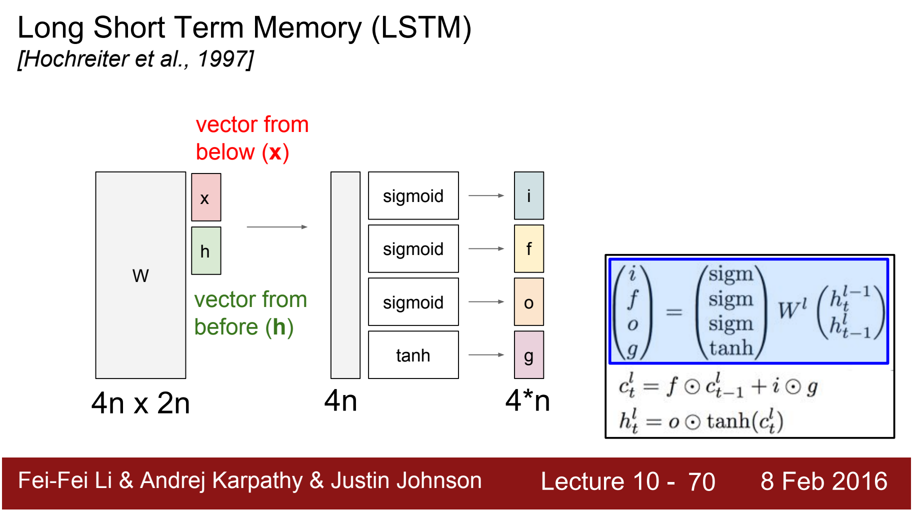
Long Short Term Memory (LSTM)
[Hochreiter et al., 1997]
vector from below (x)
x
h
W
vector from before (h)
4n x 2n
4n
sigmoid
sigmoid
sigmoid
tanh
i
f
o
g
4*n
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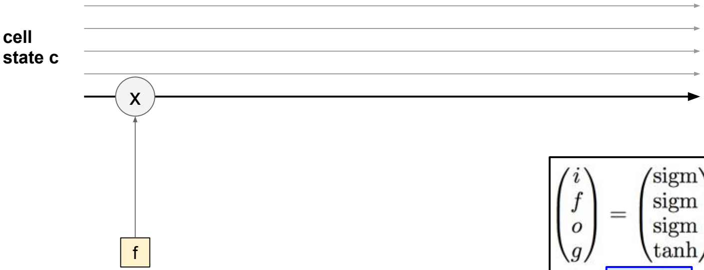
Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



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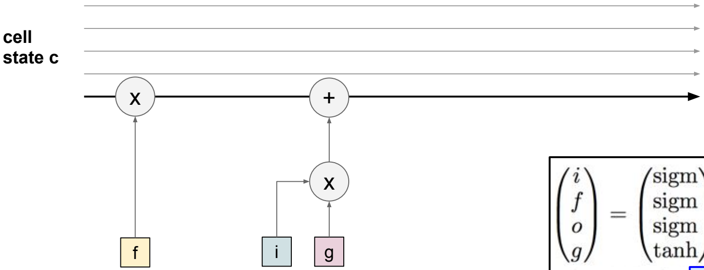

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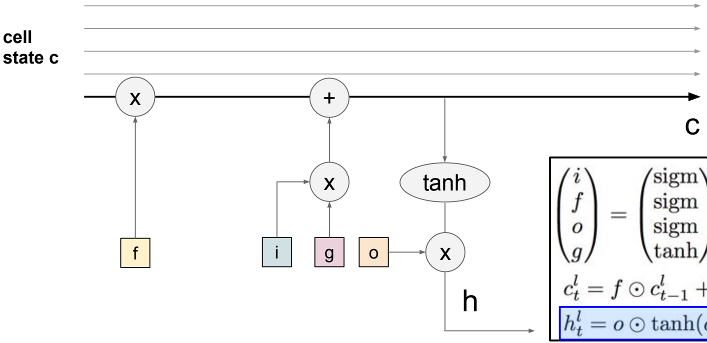

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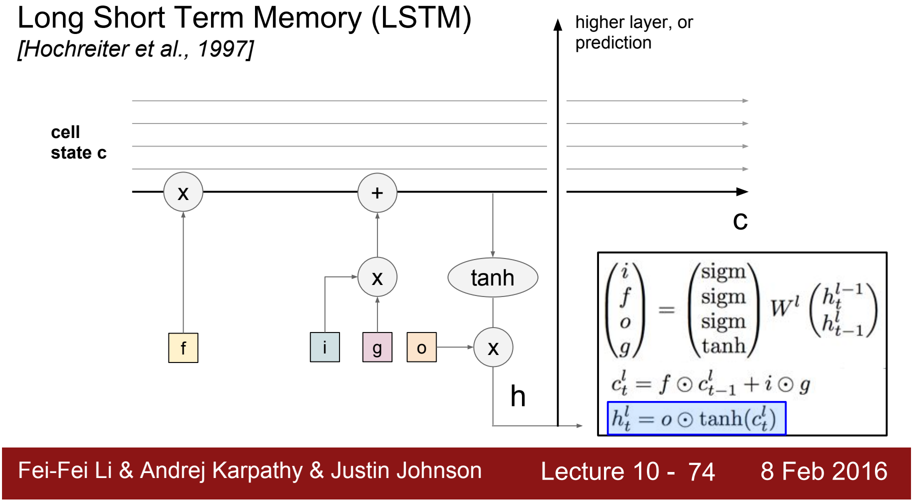

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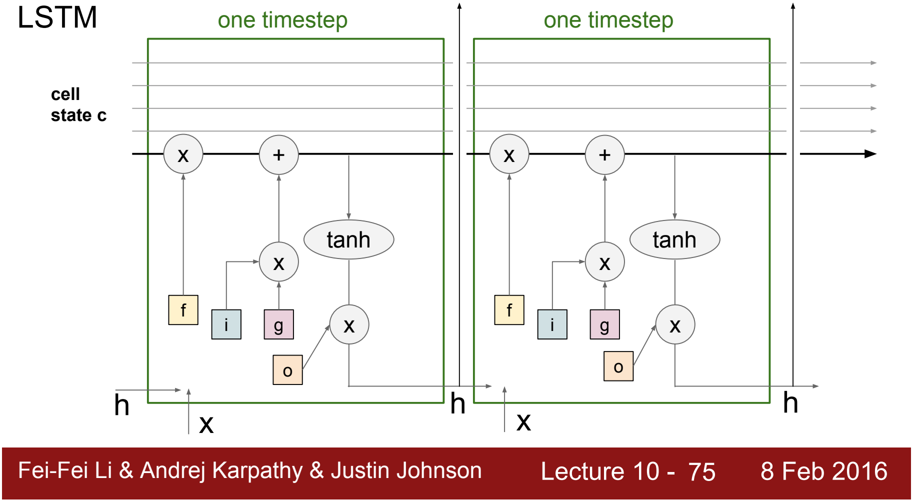
LSTM
one timestep
one timestep
cell
state c
x
+
tanh
x
x
f
i
g
o
h
x
h
x
h
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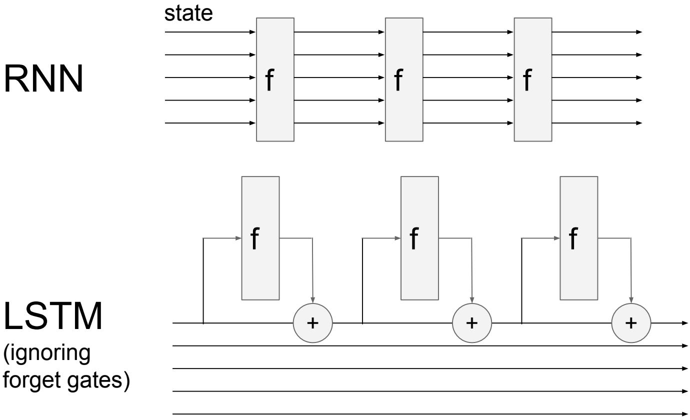

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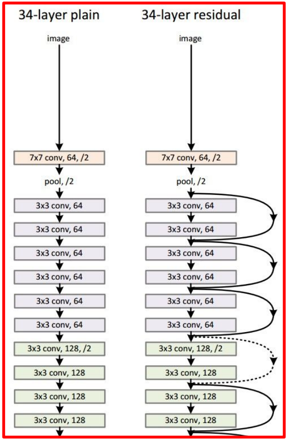
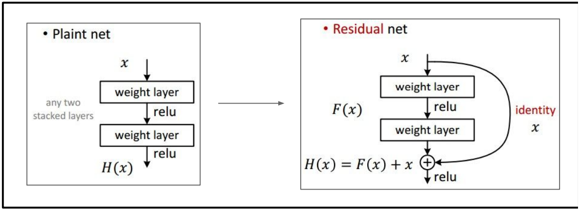
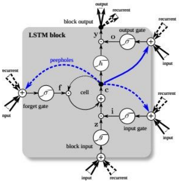


Recall: "PlainNets" vs. ResNets

ResNet is to PlainNet what LSTM is to RNN, kind of.



LSTM variants and friends



[*LSTM: A Search Space Odyssey*, Greff et al., 2015]

GRU [*Learning phrase representations using rnn encoder-decoder for statistical machine translation*, Cho et al. 2014]

$$
\begin{aligned}
r_t &= \mathrm{sigm}\left(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_{t-1} + b_{\mathrm{r}}\right) \\
z_t &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} h_{t-1} + b_{\mathrm{z}}) \\
\tilde{h}_t &= \tanh(W_{\mathrm{xh}} x_t + W_{\mathrm{hh}}(r_t \odot h_{t-1}) + b_{\mathrm{h}}) \\
h_t &= z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t
\end{aligned}
$$

[*An Empirical Exploration of Recurrent Network Architectures*, Jozefowicz et al., 2015]

MUT1:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + \tanh(x_t) + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT2:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} h_t + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

MUT3:

$$
\begin{aligned}
z &= \mathrm{sigm}(W_{\mathrm{xz}} x_t + W_{\mathrm{hz}} \tanh(h_t) + b_{\mathrm{z}}) \\
r &= \mathrm{sigm}(W_{\mathrm{xr}} x_t + W_{\mathrm{hr}} h_t + b_{\mathrm{r}}) \\
h_{t+1} &= \tanh(W_{\mathrm{hh}}(r \odot h_t) + W_{xh} x_t + b_{\mathrm{h}}) \odot z \\
&+ h_t \odot (1 - z)
\end{aligned}
$$

Modelling text: why working with characters?

- The web is composed of character strings
- Any learning method powerful enough to understand the world by reading the web ought to find it trivial to learn which strings make words (this turns out to be true, as we shall see)
- Pre-processing text to get words is a big hassle
  - What about morphemes (prefixes, suffixes etc)
  - What about subtle effects like “sn” words?
  - What about New York vs new York Minster roof?
  - What about Finnish
    - ymmärtämättömyydellänsäkään

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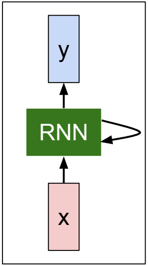
Simplest model: a first attempt

Character-level language model example

Vocabulary: [h,e,l,o]

Example training sequence: **“hello”**



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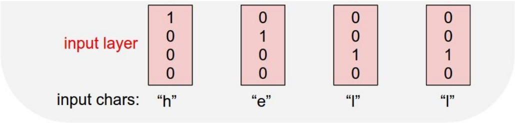

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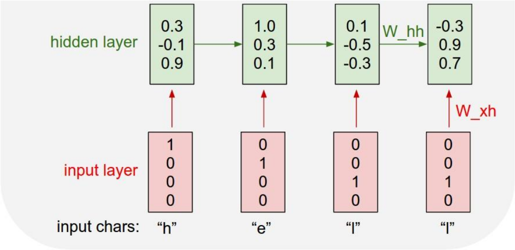

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Simplest model: a first attempt

Character-level language model example

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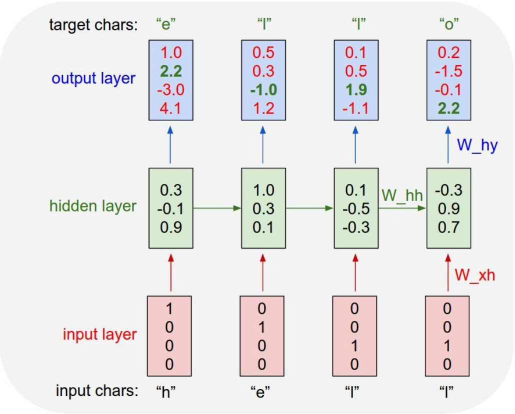

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Sampling

- Start the model with its default hidden state
- Give it a “burn-in” sequence of characters and let it update its hidden state after each character
- Then look at the probability distribution it predicts for the next character
- Pick a character randomly from that distribution and tell the net that this was the character that actually occurred
  - i.e. tell it that its guess was correct, whatever it guessed
- Continue to let it pick characters until bored

min-char-rnn.py gist: 112 lines of Python

```python
"""
Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
BSD License
"""
import numpy as np

# data I/O
data = open('input.txt', 'r').read() # should be simple plain text file
chars = list(set(data))
data_size, vocab_size = len(data), len(chars)
print 'data has %d characters, %d unique.' % (data_size, vocab_size)
char_to_ix = { ch:i for i,ch in enumerate(chars) }
ix_to_char = { i:ch for i,ch in enumerate(chars) }

# hyperparameters
hidden_size = 100 # size of hidden layer of neurons
seq_length = 25 # number of steps to unroll the RNN for
learning_rate = 1e-1

# model parameters
Wxh = np.random.randn(hidden_size, vocab_size)*0.01 # input to hidden
Whh = np.random.randn(hidden_size, hidden_size)*0.01 # hidden to hidden
Why = np.random.randn(vocab_size, hidden_size)*0.01 # hidden to output
bh = np.zeros((hidden_size, 1)) # hidden bias
by = np.zeros((vocab_size, 1)) # output bias

def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
  # backward pass: compute gradients going backwards
  dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
  dbh, dby = np.zeros_like(bh), np.zeros_like(by)
  dhnext = np.zeros_like(hs[0])
  for t in reversed(xrange(len(inputs))):
    dy = np.copy(ps[t])
    dy[targets[t]] -= 1 # backprop into y
    dWhy += np.dot(dy, hs[t].T)
    dby += dy
    dh = np.dot(Why.T, dy) + dhnext # backprop into h
    dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
    dbh += dhraw
    dWxh += np.dot(dhraw, xs[t].T)
    dWhh += np.dot(dhraw, hs[t-1].T)
    dhnext = np.dot(Whh.T, dhraw)
  for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
  return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]
```

```python
def sample(h, seed_ix, n):
  """
  sample a sequence of integers from the model
  h is memory state, seed_ix is seed letter for first time step
  """
  x = np.zeros((vocab_size, 1))
  x[seed_ix] = 1
  ixes = []
  for t in xrange(n):
    h = np.tanh(np.dot(Wxh, x) + np.dot(Whh, h) + bh)
    y = np.dot(Why, h) + by
    p = np.exp(y) / np.sum(np.exp(y))
    ix = np.random.choice(range(vocab_size), p=p.ravel())
    x = np.zeros((vocab_size, 1))
    x[ix] = 1
    ixes.append(ix)
  return ixes

n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

(https://gist.github.com/karpathy/d4dee566867f8291f086)

min-char-rnn.py gist

Data I/O

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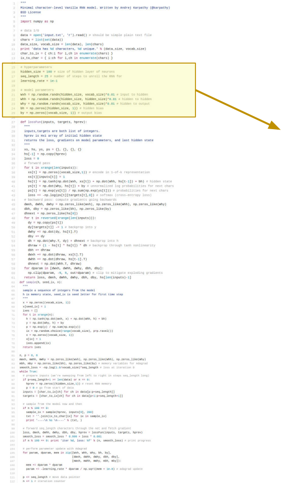
min-char-rnn.py gist



Initializations

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bh = np.zeros((hidden_size, 1)) # hidden bias
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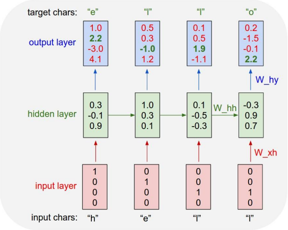
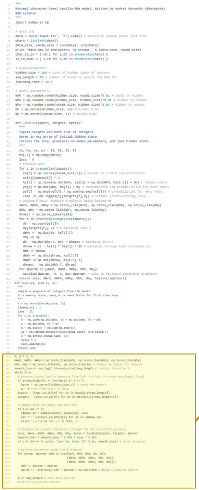
```

recall:



min-char-rnn.py gist

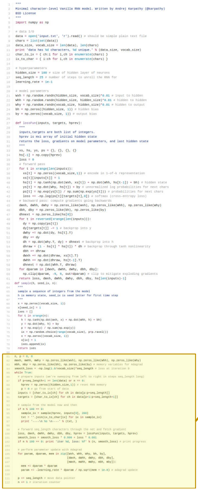


Main loop

```python
n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

Example: simple character-level language model

min-char-rnn.py gist



Main loop

```python
n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

min-char-rnn.py gist

Main loop

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mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
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    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

min-char-rnn.py gist

Main loop

```python
n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
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                                [dWxh, dWhh, dWhy, dbh, dby],
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    mem += dparam * dparam
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  p += seq_length # move data pointer
  n += 1 # iteration counter
```
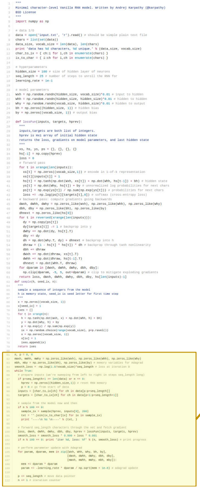

min-char-rnn.py gist



Main loop

```python
n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
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    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```
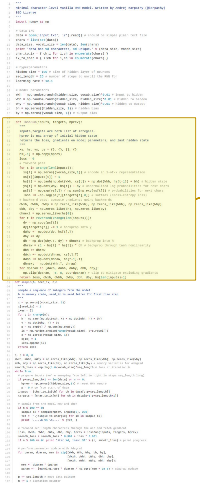

min-char-rnn.py gist

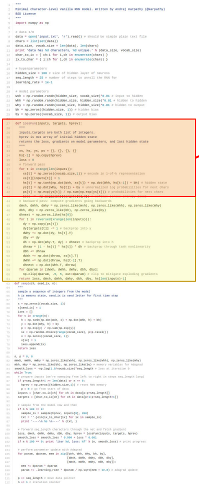


Loss function

- forward pass (compute loss)
- backward pass (compute param gradient)

```python
def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
  # backward pass: compute gradients going backwards
  dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
  dbh, dby = np.zeros_like(bh), np.zeros_like(by)
  dhnext = np.zeros_like(hs[0])
  for t in reversed(xrange(len(inputs))):
    dy = np.copy(ps[t])
    dy[targets[t]] -= 1 # backprop into y
    dWhy += np.dot(dy, hs[t].T)
    dby += dy
    dh = np.dot(Why.T, dy) + dhnext # backprop into h
    dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
    dbh += dhraw
    dWxh += np.dot(dhraw, xs[t].T)
    dWhh += np.dot(dhraw, hs[t-1].T)
    dhnext = np.dot(Whh.T, dhraw)
  for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
  return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]
```

min-char-rnn.py gist



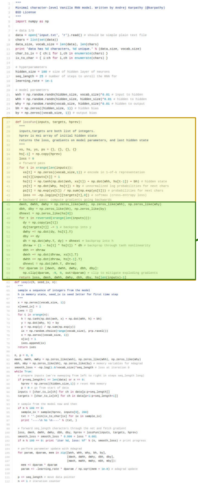
```python
def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
```

$$h_t = \tanh(W_{hh} h_{t-1} + W_{xh} x_t)$$

$$y_t = W_{hy} h_t$$

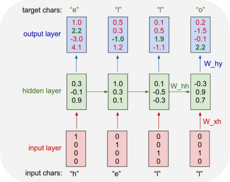
Softmax classifier

min-char-rnn.py gist



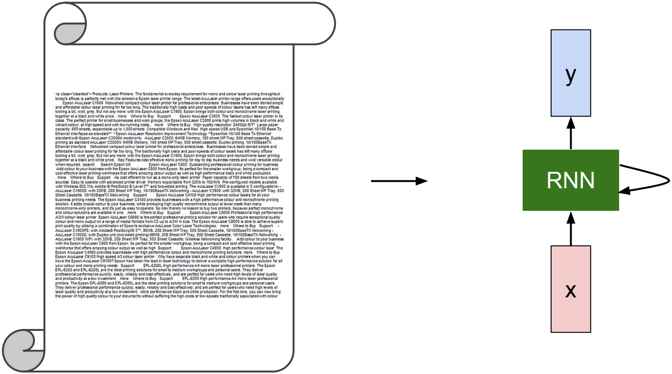
```
# backward pass: compute gradients going backwards
dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
dbh, dby = np.zeros_like(bh), np.zeros_like(by)
dhnext = np.zeros_like(hs[0])
for t in reversed(xrange(len(inputs))):
  dy = np.copy(ps[t])
  dy[targets[t]] -= 1 # backprop into y
  dWhy += np.dot(dy, hs[t].T)
  dby += dy
  dh = np.dot(Why.T, dy) + dhnext # backprop into h
  dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
  dbh += dhraw
  dWxh += np.dot(dhraw, xs[t].T)
  dWhh += np.dot(dhraw, hs[t-1].T)
  dhnext = np.dot(Whh.T, dhraw)
for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
  np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]
```

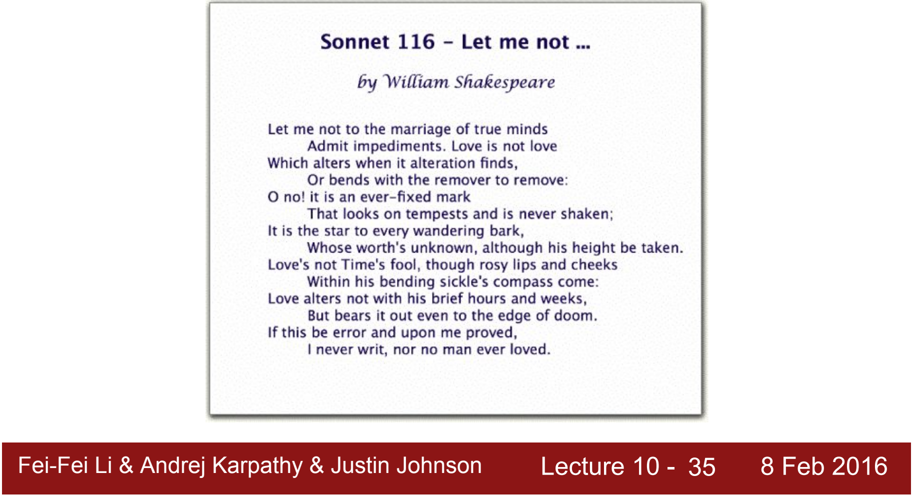
recall:



min-char-rnn.py gist

```
def sample(h, seed_ix, n):
  """
  sample a sequence of integers from the model
  h is memory state, seed_ix is seed letter for first time step
  """
  x = np.zeros((vocab_size, 1))
  x[seed_ix] = 1
  ixes = []
  for t in xrange(n):
    h = np.tanh(np.dot(Wxh, x) + np.dot(Whh, h) + bh)
    y = np.dot(Why, h) + by
    p = np.exp(y) / np.sum(np.exp(y))
    ix = np.random.choice(range(vocab_size), p=p.ravel())
    x = np.zeros((vocab_size, 1))
    x[ix] = 1
    ixes.append(ix)
  return ixes
```


y
RNN
x



Sonnet 116 – Let me not ...

by William Shakespeare

Let me not to the marriage of true minds
Admit impediments. Love is not love
Which alters when it alteration finds,
Or bends with the remover to remove:
O no! it is an ever-fixed mark
That looks on tempests and is never shaken;
It is the star to every wandering bark,
Whose worth's unknown, although his height be taken.
Love's not Time's fool, though rosy lips and cheeks
Within his bending sickle's compass come:
Love alters not with his brief hours and weeks,
But bears it out even to the edge of doom.
If this be error and upon me proved,
I never writ, nor no man ever loved.

Fei-Fei Li & Andrej Karpathy & Justin Johnson Lecture 10 - 35 8 Feb 2016

at first:

```
tyntd-iafhatawiaoihrdemot  lytdws  e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e
plia tklrgd t o idoe ns,smtt   h ne etie h,hregtrs nigtike,aoaenns lng
```

↓ train more

```
"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."
```

↓ train more

```
Aftair fall unsuch that the hall for Prince Velzonski's that me of
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort
how, and Gogition is so overelical and ofter.
```

↓ train more

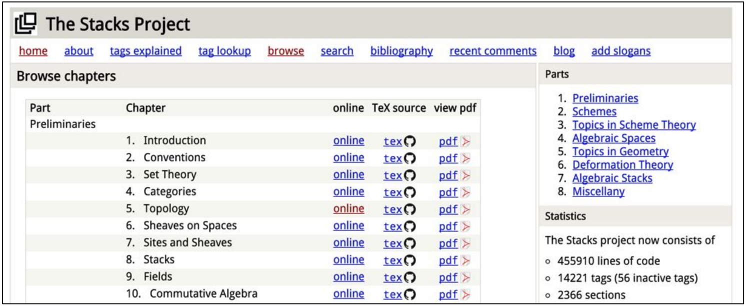
```
"Why do what that day," replied Natasha, and wishing to himself the fact the
princess, Princess Mary was easier, fed in had offened him.
Pierre aking his soul came to the packs and drove up his father-in-law women.
```

```
PANDARUS:
Alas, I think he shall be come approached and the day
When little srain would be attain'd into being never fed,
And who is but a chain and subjects of his death,
I should not sleep.

Second Senator:
They are away this miseries, produced upon my soul,
Breaking and strongly should be buried, when I perish
The earth and thoughts of many states.

DUKE VINCENTIO:
Well, your wit is in the care of side and that.

Second Lord:
They would be ruled after this chamber, and
my fair nues begun out of the fact, to be conveyed,
Whose noble souls I'll have the heart of the wars.

Clown:
Come, sir, I will make did behold your worship.

VIOLA:
I'll drink it.
```

```
VIOLA:
Why, Salisbury must find his flesh and thought
That which I am not aps, not a man and in fire,
To show the reining of the raven and the wars
To grace my hand reproach within, and not a fair are hand,
That Caesar and my goodly father's world;
When I was heaven of presence and our fleets,
We spare with hours, but cut thy council I am great,
Murdered and by thy master's ready there
My power to give thee but so much as hell:
Some service in the noble bondman here,
Would show him to her wine.

KING LEAR:
O, if you were a feeble sight, the courtesy of your law,
Your sight and several breath, will wear the gods
With his heads, and my hands are wonder'd at the deeds,
So drop upon your lordship's head, and your opinion
Shall be against your honour.
```

Fei-Fei Li & Andrej Karpathy & Justin Johnson   Lecture 10 - 37   8 Feb 2016

open source textbook on algebraic geometry


The Stacks Project

home about tags explained tag lookup browse search bibliography recent comments blog add slogans

Browse chapters

Part	Chapter	online	TeX source	view pdf
Preliminaries				
	1. Introduction	online	tex	pdf
	2. Conventions	online	tex	pdf
	3. Set Theory	online	tex	pdf
	4. Categories	online	tex	pdf
	5. Topology	online	tex	pdf
	6. Sheaves on Spaces	online	tex	pdf
	7. Sites and Sheaves	online	tex	pdf
	8. Stacks	online	tex	pdf
	9. Fields	online	tex	pdf
	10. Commutative Algebra	online	tex	pdf

Parts

1. Preliminaries
2. Schemes
3. Topics in Scheme Theory
4. Algebraic Spaces
5. Topics in Geometry
6. Deformation Theory
7. Algebraic Stacks
8. Miscellany

Statistics

The Stacks project now consists of

- 455910 lines of code
- 14221 tags (56 inactive tags)
- 2366 sections

Latex source

For $\bigoplus_{n=1,\ldots,m}$ where $\mathcal{L}_{m_\bullet} = 0$, hence we can find a closed subset $\mathcal{H}$ in $\mathcal{H}$ and any sets $\mathcal{F}$ on X, U is a closed immersion of S, then $U \to T$ is a separated algebraic space.

Proof. Proof of (1). It also start we get

$$S = \mathrm{Spec}(R) = U \times_X U \times_X U$$

and the comparicoly in the fibre product covering we have to prove the lemma generated by $\coprod Z \times_U U \to V$. Consider the maps M along the set of points Sch_{fppf} and $U \to U$ is the fibre category of S in U in Section, ?? and the fact that any U affine, see Morphisms, Lemma ??. Hence we obtain a scheme S and any open subset $W \subset U$ in $Sh(G)$ such that $\mathrm{Spec}(R') \to S$ is smooth or an

$$U = \bigcup U_i \times_{S_i} U_i$$

which has a nonzero morphism we may assume that f_i is of finite presentation over S. We claim that $\mathcal{O}_{X,x}$ is a scheme where $x, x', s'' \in S'$ such that $\mathcal{O}_{X,x'} \to \mathcal{O}'_{X',x'}$ is separated. By Algebra, Lemma ?? we can define a map of complexes $\mathrm{GL}_{S'}(x'/S'')$ and we win. □

To prove study we see that $\mathcal{F}|_U$ is a covering of $\mathcal{X}'$, and $\mathcal{T}_i$ is an object of $\mathcal{F}_{X/S}$ for $i > 0$ and $\mathcal{F}_p$ exists and let $\mathcal{F}_i$ be a presheaf of $\mathcal{O}_X$-modules on $\mathcal{C}$ as a $\mathcal{F}$-module. In particular $\mathcal{F} = U/\mathcal{F}$ we have to show that

$$\widetilde{M}^\bullet = \mathcal{I}^\bullet \otimes_{\mathrm{Spec}(k)} \mathcal{O}_{S,s} - i_X^{-1}\mathcal{F})$$

is a unique morphism of algebraic stacks. Note that

$$\mathrm{Arrows} = (Sch/S)^{opp}_{fppf}, (Sch/S)_{fppf}$$

and

$$V = \Gamma(S, \mathcal{O}) \longmapsto (U, \mathrm{Spec}(A))$$

is an open subset of X. Thus U is affine. This is a continuous map of X is the inverse, the groupoid scheme S.

Proof. See discussion of sheaves of sets. □

The result for prove any open covering follows from the less of Example ??. It may replace S by $X_{spaces,\acute{e}tale}$ which gives an open subspace of X and T equal to S_{Zar}, see Descent, Lemma ??. Namely, by Lemma ?? we see that R is geometrically regular over S.

Lemma 0.1. *Assume (3) and (3) by the construction in the description.*

Suppose $X = \lim |X|$ (by the formal open covering X and a single map $\underline{Proj}_X(\mathcal{A}) =$ $\mathrm{Spec}(B)$ over U compatible with the complex

$$Set(\mathcal{A}) = \Gamma(X, \mathcal{O}_{X,\mathcal{O}_X}).$$

When in this case of to show that $\mathcal{Q} \to \mathcal{C}_{Z/X}$ is stable under the following result in the second conditions of (1), and (3). This finishes the proof. By Definition ?? (without element is when the closed subschemes are catenary. If T is surjective we may assume that T is connected with residue fields of S. Moreover there exists a closed subspace $Z \subset X$ of X where U in X' is proper (some defining as a closed subset of the uniqueness it suffices to check the fact that the following theorem

(1) *f is locally of finite type. Since $S = \mathrm{Spec}(R)$ and $Y = \mathrm{Spec}(R)$.*

Proof. This is form all sheaves of sheaves on X. But given a scheme U and a surjective étale morphism $U \to X$. Let $U \cap U = \coprod_{i=1,\ldots,n} U_i$ be the scheme X over S at the schemes $X_i \to X$ and $U = \lim_i X_i$. □

The following lemma surjective restrocomposes of this implies that $\mathcal{F}_{x_0} = \mathcal{F}_{x_0} = \mathcal{F}_{\mathcal{X},\ldots,0}$.

Lemma 0.2. *Let X be a locally Noetherian scheme over S, $E = \mathcal{F}_{X/S}$. Set $\mathcal{I} = \mathcal{J}_1 \subset \mathcal{I}'_n$. Since $\mathcal{I}^n \subset \mathcal{I}^n$ are nonzero over $i_0 \leq \mathfrak{p}$ is a subset of $\mathcal{J}_{n,0} \circ \overline{A}_2$ works.*

Lemma 0.3. *In Situation ??. Hence we may assume $\mathfrak{q}' = 0$.*

Proof. We will use the property we see that $\mathfrak{p}$ is the mext functor (??). On the other hand, by Lemma ?? we see that

$$D(\mathcal{O}_{X'}) = \mathcal{O}_X(D)$$

where K is an F-algebra where δ_{n+1} is a scheme over S. □

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{\acute{e}tale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

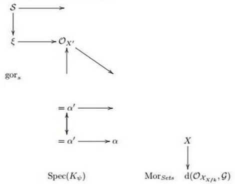
be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

This since $\mathcal{F} \in \mathcal{F}$ and $x \in \mathcal{G}$ the diagram



is a limit. Then $\mathcal{G}$ is a finite type and assume S is a flat and $\mathcal{F}$ and $\mathcal{G}$ is a finite type f_*. This is of finite type diagrams, and

- the composition of $\mathcal{G}$ is a regular sequence,
- $\mathcal{O}_{X'}$ is a sheaf of rings.

□

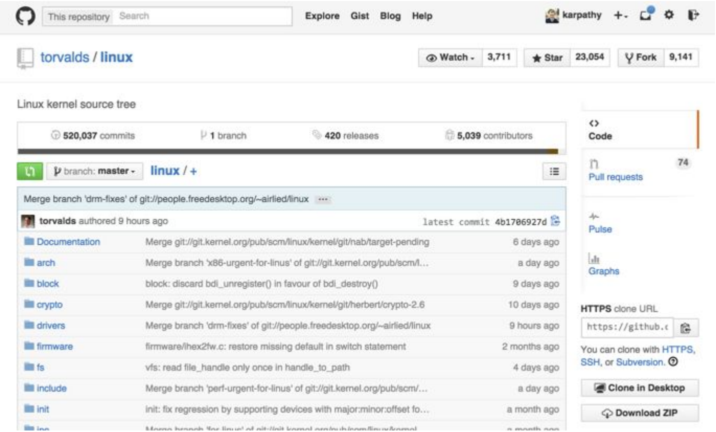
Proof. We have see that $X = \mathrm{Spec}(R)$ and $\mathcal{F}$ is a finite type representable by algebraic space. The property $\mathcal{F}$ is a finite morphism of algebraic stacks. Then the cohomology of X is an open neighbourhood of U. □

Proof. This is clear that $\mathcal{G}$ is a finite presentation, see Lemmas ??.
A *reduced above* we conclude that U is an open covering of $\mathcal{C}$. The functor $\mathcal{F}$ is a "field

$$\mathcal{O}_{X,x} \longrightarrow \mathcal{F}_{\overline{x}} \ \text{-1}(\mathcal{O}_{X_{\acute{e}tale}}) \longrightarrow \mathcal{O}_{X_\ell}^{-1}\mathcal{O}_{X_\lambda}(\mathcal{O}_{X_\eta}^{\overline{v}})$$

is an isomorphism of covering of $\mathcal{O}_{X_i}$. If $\mathcal{F}$ is the unique element of $\mathcal{F}$ such that X is an isomorphism.
The property $\mathcal{F}$ is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme $\mathcal{O}_X$-algebra with $\mathcal{F}$ are opens of finite type over S.
If $\mathcal{F}$ is a scheme theoretic image points. □

If $\mathcal{F}$ is a finite direct sum $\mathcal{O}_{X_\lambda}$ is a closed immersion, see Lemma ??. This is a sequence of $\mathcal{F}$ is a similar morphism.



Fei-Fei Li & Andrej Karpathy & Justin Johnson Lecture 10 - 41 8 Feb 2016

```c
static void do_command(struct seq_file *m, void *v)
{
  int column = 32 << (cmd[2] & 0x80);
  if (state)
    cmd = (int)(int_state ^ (in_8(&ch->ch_flags) & Cmd) ? 2 : 1);
  else
    seq = 1;
  for (i = 0; i < 16; i++) {
    if (k & (1 << 1))
      pipe = (in_use & UMXTHREAD_UNCCA) +
        ((count & 0x00000000fffffff8) & 0x000000f) << 8;
    if (count == 0)
      sub(pid, ppc_md.kexec_handle, 0x20000000);
    pipe_set_bytes(i, 0);
  }
  /* Free our user pages pointer to place camera if all dash */
  subsystem_info = &of_changes[PAGE_SIZE];
  rek_controls(offset, idx, &soffset);
  /* Now we want to deliberately put it to device */
  control_check_polarity(&context, val, 0);
  for (i = 0; i < COUNTER; i++)
    seq_puts(s, "policy ");
}
```

Generated C code

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```
/*
 * Copyright (c) 2006-2010, Intel Mobile Communications.  All rights reserved.
 *
 *   This program is free software; you can redistribute it and/or modify it
 * under the terms of the GNU General Public License version 2 as published by
 * the Free Software Foundation.
 *
 *        This program is distributed in the hope that it will be useful,
 * but WITHOUT ANY WARRANTY; without even the implied warranty of
 *   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
 *
 *  GNU General Public License for more details.
 *
 *   You should have received a copy of the GNU General Public License
 *    along with this program; if not, write to the Free Software Foundation,
 *  Inc., 675 Mass Ave, Cambridge, MA 02139, USA.
 */

#include <linux/kexec.h>
#include <linux/errno.h>
#include <linux/io.h>
#include <linux/platform_device.h>
#include <linux/multi.h>
#include <linux/ckevent.h>

#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>
```

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#include <asm/io.h>
#include <asm/prom.h>
#include <asm/e820.h>
#include <asm/system_info.h>
#include <asm/setew.h>
#include <asm/pgproto.h>

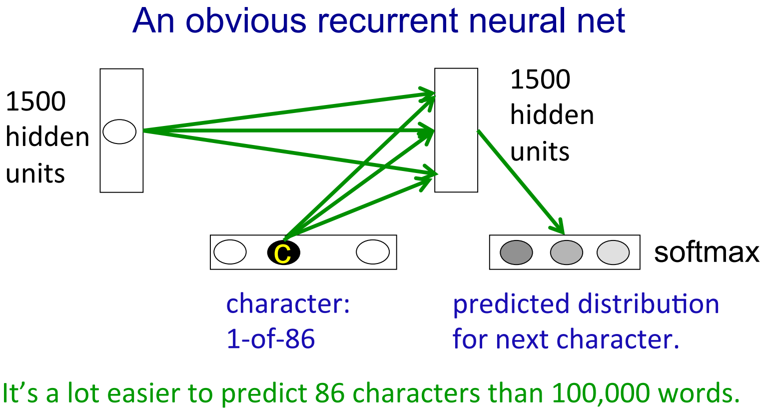
#define REG_PG    vesa_slot_addr_pack
#define PFM_NOCOMP  AFSR(0, load)
#define STACK_DDR(type)     (func)

#define SWAP_ALLOCATE(nr)     (e)
#define emulate_sigs()  arch_get_unaligned_child()
#define access_rw(TST)  asm volatile("movd %%esp, %0, %3" : : "r" (0));   \
  if (__type & DO_READ)

static void stat_PC_SEC __read_mostly offsetof(struct seq_argsqueue, \
          pC>[1]);

static void
os_prefix(unsigned long sys)
{
#ifdef CONFIG_PREEMPT
  PUT_PARAM_RAID(2, sel) = get_state_state();
  set_pid_sum((unsigned long)state, current_state_str(),
           (unsigned long)-1->lr_full; low;
}
```

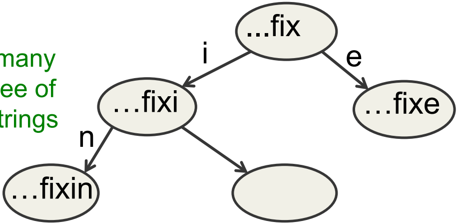
Ideal model?


An obvious recurrent neural net
1500
hidden
units
1500
hidden
units
c
softmax
character:
1-of-86
predicted distribution
for next character.
It's a lot easier to predict 86 characters than 100,000 words.

A slight tweak: Ideal tree model

An ideal model considers all previous input characters and the current character

There are exponentially many nodes in the tree of all character strings of length N.



In an RNN, each node is a hidden state vector. The next character must transform this to a new node.

- The next hidden representation needs to depend on the conjunction of the current character and the current hidden representation
  - We expect under each hidden state vector and each current character, we should have a different transition matrix. The earlier simple model tried to capture this but is kind of indirect

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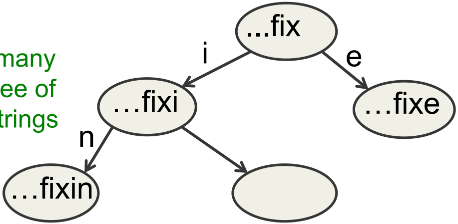

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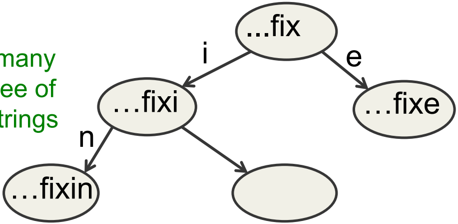

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Multiplicative connections

- We may prepare a different transition matrix for each input
  - But this requires 86x1500x1500 parameters (let say we have 1500 hidden variables)
  - And this could make the net overfit
- Can we achieve the same kind of multiplicative interaction using fewer parameters?
  - We want a different transition matrix for each of the 86 characters, but we want these 86 character-specific weight matrices to share parameters (the characters 9 and 8 should have similar matrices)

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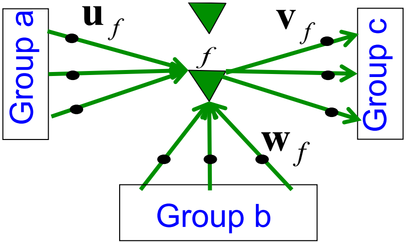
Using factors to implement multiplicative interactions

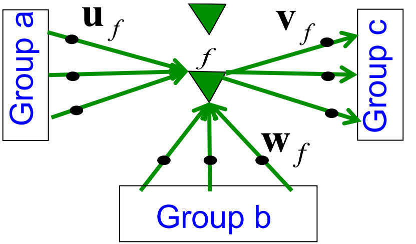


Vector input to group c:

$$c_f = \underbrace{(b^T w_f)}_{\text{Scalar input from group } b} \underbrace{(a^T u_f)}_{\text{Scalar input from group } a} v_f$$

- We can get groups a and b to interact multiplicatively by using "factors"
  - Each factor first computes a weighted sum for each of its input groups
  - Then it sends the product of the weighted sums to its output group

Using factors to implement multiplicative interactions



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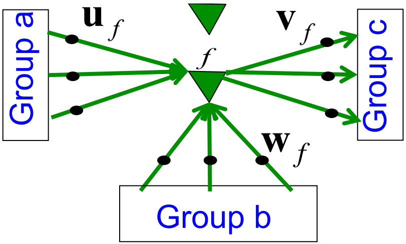

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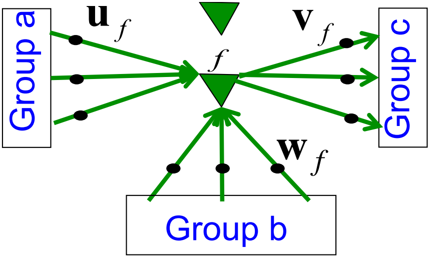
Using factors to implement a set of basis matrices

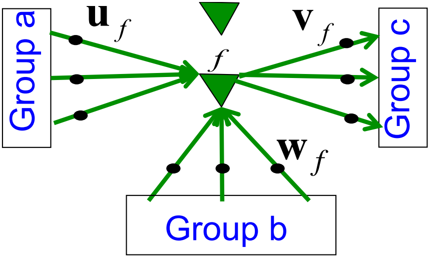


$$c_f = (b^T w_f)(a^T u_f) v_f = (b^T w_f) v_f (u_f^T a) = \underbrace{(b^T w_f)}_{\text{scalar coefficient}} \underbrace{(v_f u_f^T)}_{\text{outer product transition matrix with rank 1}} a$$

- We can think about factors another way:
  - Each factor defines a rank 1 transition matrix from a to c

$$c = \left(\sum_f (b^T w_f)(v_f u_f^T) \right) a$$

Using factors to implement a set of basis matrices



$$
\begin{aligned}
c_f &= (b^T w_f)(a^T u_f) v_f \\
&= (b^T w_f) v_f (u_f^T a) \\
&= \underbrace{(b^T w_f)}_{\text{scalar coefficient}} \underbrace{(v_f u_f^T)}_{\text{outer product transition matrix with rank 1}} a
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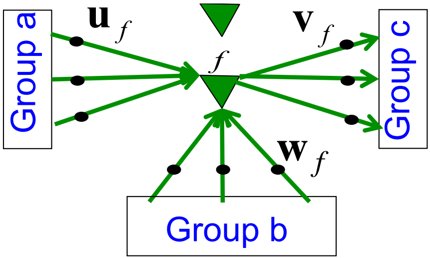

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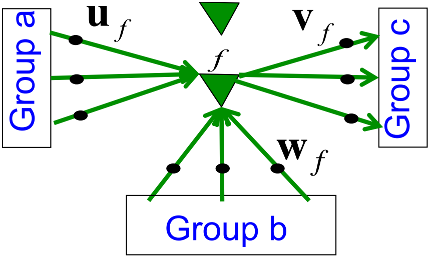

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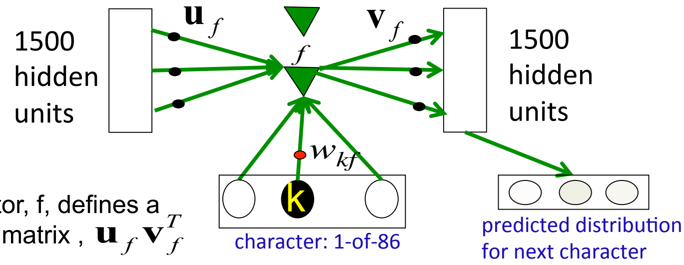
$$

Using 3-way factors to allow a character to create a whole transition matrix



Each factor, f, defines a rank one matrix , $\mathbf{u}_f \mathbf{v}_f^T$

Each character, k, determines a gain w_{kf} for each of these matrices.

Some note on optimization

- To optimize efficiently, they use Hessian-free (HF) method to minimize the cost
- HF is a second order method similar to Newton methods and LBFGS that take advantage of the curvature (Hessian) matrix
- In the HF method, they make an approximation to the curvature matrix and then minimize the error using conjugate gradient method. Then they make another approximation to the curvature matrix and minimize again

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Conjugate gradient

- There is an alternative to going to the minimum in one step by multiplying by the inverse of the curvature matrix
- Use a sequence of steps each of which finds the minimum along one direction
- Make sure that each new direction is "conjugate" to the previous directions so you do not mess up the minimization you already did.
  - "conjugate" means that as you go in the new direction, you do not change the gradients in the previous directions

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Training the model

- Ilya Sutskever used 5 million strings of 100 characters taken from wikipedia. For each string he starts predicting at the $11th$ character
- Using the HF optimizer, it took a month on a GPU board to get a really good model (back in 2011) text

Result

He was elected President during the Revolutionary War and forgave Opus Paul at Rome. The regime of his crew of England, is now Arab women's icons in and the demons that use something between the characters' sisters in lower coil trains were always operated on the line of the ephemerable street, respectively, the graphic or other facility for deformation of a given proportion of large segments at RTUS). The B every chord was a "strongly cold internal palette pour even the white blade."

Result: some completions produced by the model

- Sheila thrunges (most frequent)
- People thrunge (most frequent next character is space)
- Shiela, Thrungelini del Rey (first try)
- The meaning of life is literary recognition. (6 th try)
- The meaning of life is the tradition of the ancient human reproduction: it is less favorable to the good boy for when to remove her bigger. (one of the first 10 tries for a model trained for longer)

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Result: what does it know?

- It knows a huge number of words and a lot about proper names, dates, and numbers
- It is good at balancing quotes and brackets
  - It can count brackets: none, one, many
- It knows a lot about syntax but its very hard to pin down exactly what grammar it actually "knows"
- It knows a lot of weak semantic associations
  - E.g. it knows Plato is associated with Wittgenstein and cabbage is associated with vegetable

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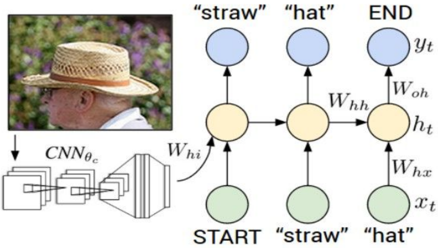
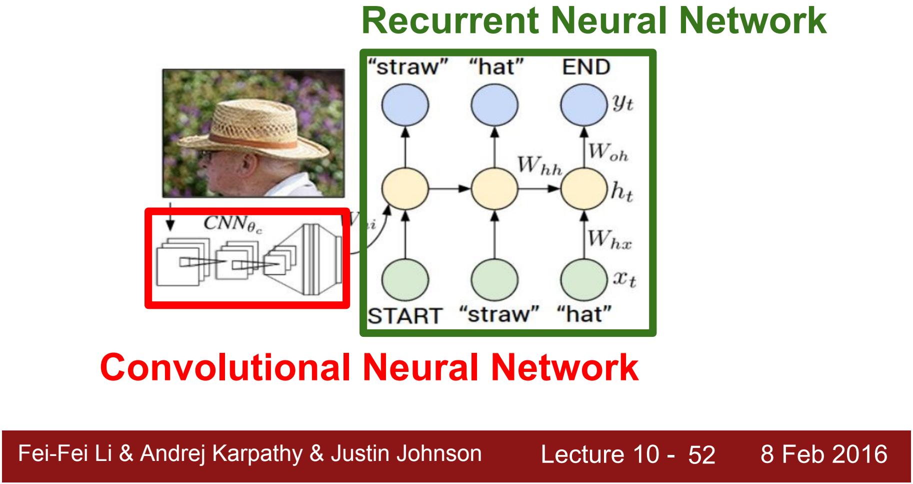
RNNs for predicting the next word

- Tomas Mikolov and his collaborators have recently trained quite large RNNs on quite large training sets using backprop through time (BPTT)
  - They do better than feed-forward neural nets
  - They do better than the best other models
  - They do even better when averaged with other models
- RNNs require much less training data to reach the same level of performance as other models
- RNNs improve faster than other methods as the dataset gets bigger
  - This is going to make them very hard to beat

Image Captioning



Explain Images with Multimodal Recurrent Neural Networks, Mao et al.
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei
Show and Tell: A Neural Image Caption Generator, Vinyals et al.
Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.
Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick


Recurrent Neural Network
"straw"
"hat"
END
y_t
W_{oh}
W_{hh}
h_t
CNN_{θ_c}
W_{hx}
x_t
START
"straw"
"hat"
Convolutional Neural Network
Fei-Fei Li & Andrej Karpathy & Justin Johnson
Lecture 10 - 52
8 Feb 2016

test image


image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
FC-1000
softmax

test image


image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096
FC-1000
softmax
X

test image

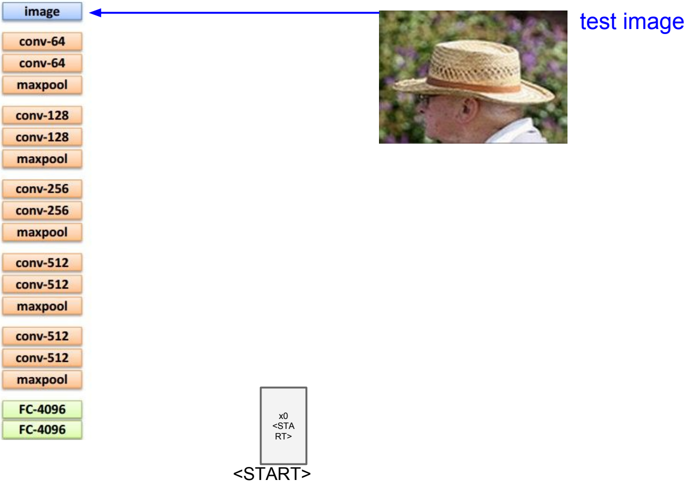


image
conv-64
conv-64
maxpool
conv-128
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conv-512
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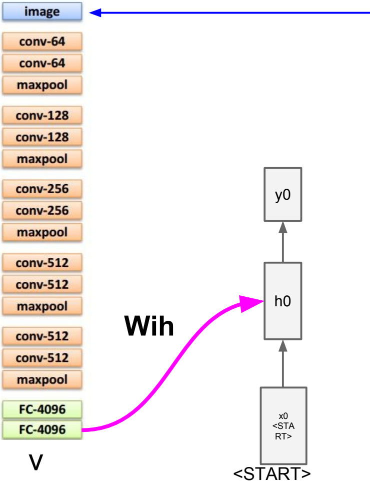
FC-4096
FC-4096
test image
x0
<STA
RT>
<START>



test image

before:

h = tanh(Wxh * x + Whh * h)

now:

h = tanh(Wxh * x + Whh * h **+ Wih * v**)

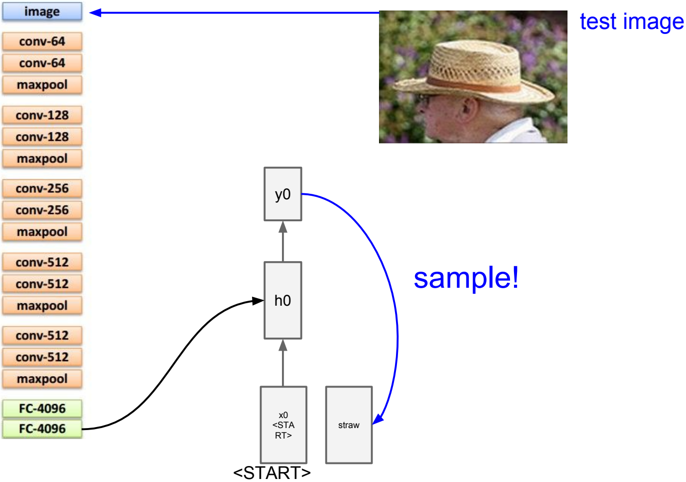


image
conv-64
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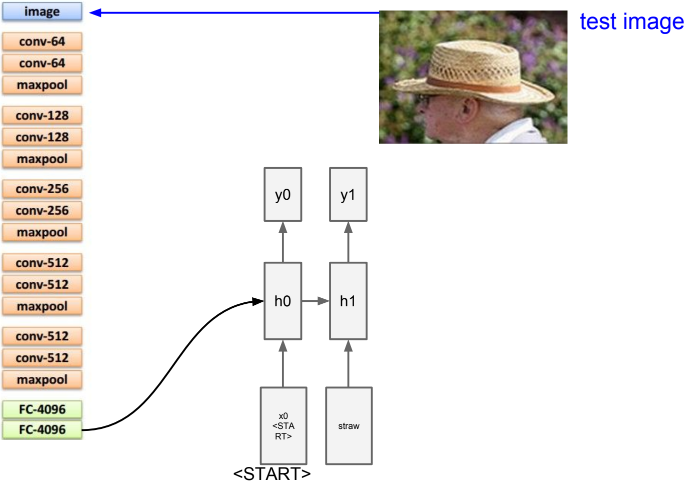
maxpool
FC-4096
FC-4096
test image
y0
h0
sample!
x0
<START>
straw
<START>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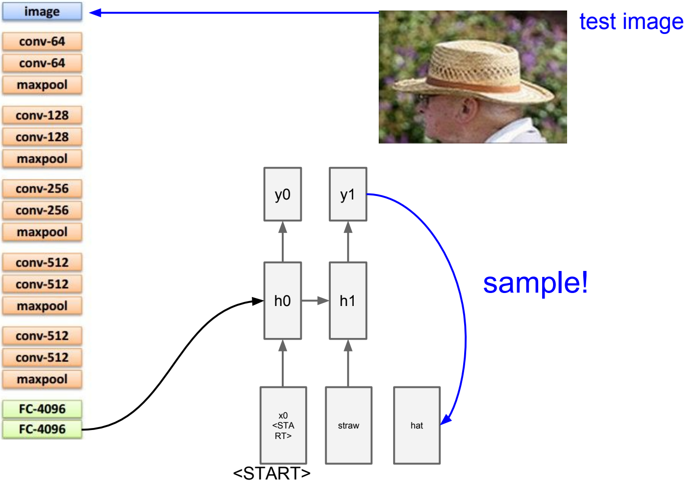
FC-4096
FC-4096
test image
y0
y1
h0
h1
x0 <STA RT>
straw
<START>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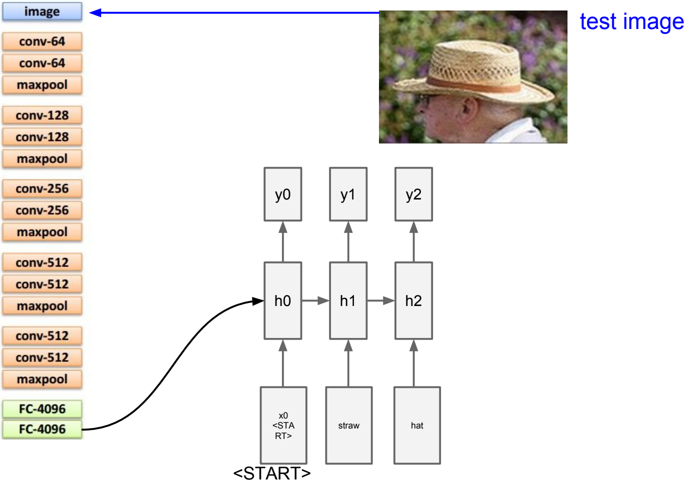
maxpool
FC-4096
FC-4096
test image
y0
y1
h0
h1
sample!
x0 <START>
straw
hat
<START>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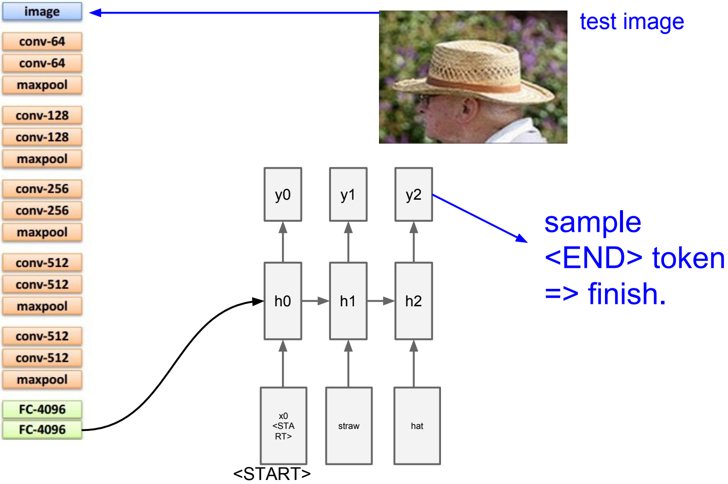
FC-4096
FC-4096
test image
y0
y1
y2
h0
h1
h2
x0 <START>
straw
hat
<START>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conv-512
maxpool
FC-4096
FC-4096
test image
y0
y1
y2
h0
h1
h2
x0 <STA RT>
straw
hat
<START>
sample <END> token => finish.

Image Sentence Datasets

a man riding a bike on a dirt path through a forest.
bicyclist raises his fist as he rides on desert dirt trail.
this dirt bike rider is smiling and raising his fist in triumph.
a man riding a bicycle while pumping his fist in the air.
a mountain biker pumps his fist in celebration.

Microsoft COCO
[Tsung-Yi Lin et al. 2014]
mscoco.org

currently:
~120K images
~5 sentences each

"man in black shirt is playing guitar."

"construction worker in orange safety vest is working on road."

"two young girls are playing with lego toy."

"boy is doing backflip on wakeboard."

"man in black shirt is playing guitar."

"construction worker in orange safety vest is working on road."

"two young girls are playing with lego toy."

"boy is doing backflip on wakeboard."

"a young boy is holding a baseball bat."

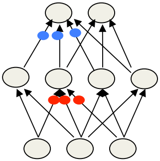
"a cat is sitting on a couch with a remote control."

"a woman holding a teddy bear in front of a mirror."

"a horse is standing in the middle of a road."

The key idea of echo state networks (perceptrons again?)

- A very simple way to learn a feedforward network is to make the early layers random and fixed.
- Then we just learn the last layer which is a linear model that uses the transformed inputs to predict the target outputs.
  - A big random expansion of the input vector can help.



- The equivalent idea for RNNs is to fix the input→hidden connections and the hidden→hidden connections at random values and only learn the hidden→output connections.
  - The learning is then very simple (assuming linear output units).
  - Its important to set the random connections very carefully so the RNN does not explode or die.

How to set random connections in echo state networks

- Set the hidden→hidden weights so that the intensity of activity stays about the same after each iteration
  - Set the largest eigenvalue to 1
  - This allows the input to echo around the network for a long time
- Use sparse connectivity (i.e. set most of the weights to zero)
  - This creates lots of loosely coupled oscillators
- Choose the scale of the input→hidden connections very carefully
  - They need to drive the loosely coupled oscillators without wiping out the information from the past that they already contain
- The learning is so fast that we can try many different scales for the input→hidden weights and sparsenesses
  - This is often necessary

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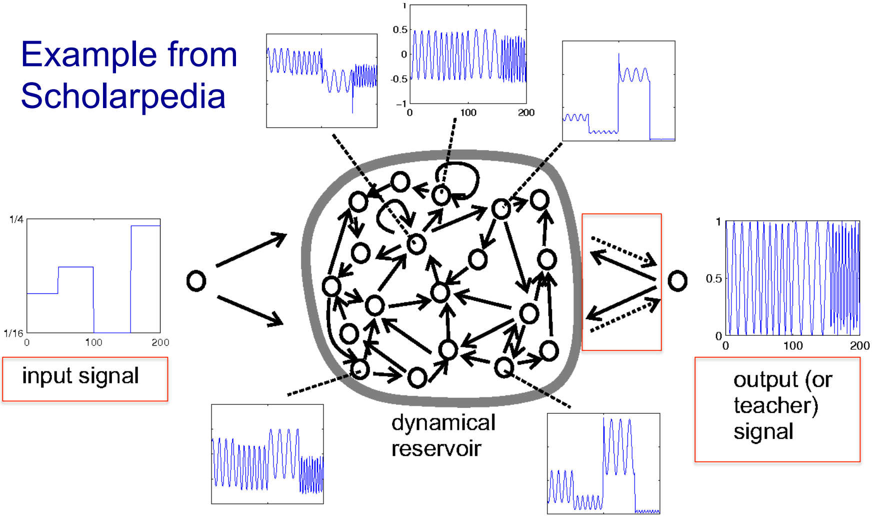
A simple example of an echo state network

INPUT SEQUENCE A real-valued time-varying value that specifies the frequency of a sine wave

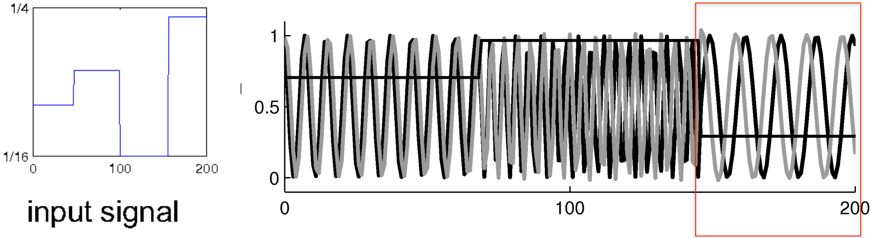
TARGET OUTPUT SEQUENCE A sine wave with the currently specified frequency

LEARNING METHOD Fit a linear model that takes the states of the hidden units as input and produces a single scalar output

Example from Scholarpedia



The target and predicted outputs after learning



Beyond echo state networks

- Good aspects of ESNs: Echo state networks can be trained very fast because they just fit a linear model
- They demonstrate that it is very important to initialize weights sensibly
- They can do impressive modeling of one-dimensional time-series
  - but they cannot compete seriously for high-dimensional data like pre-processed speech

- Bad aspects of ESNs: They need many more hidden units for a given task than an RNN that learns the hidden→hidden weights
- Ilya Sutskever (2012) has shown that if the weights are initialized using the ESN methods, RNNs can be trained very effectively
  - He uses rmsprop with momentum

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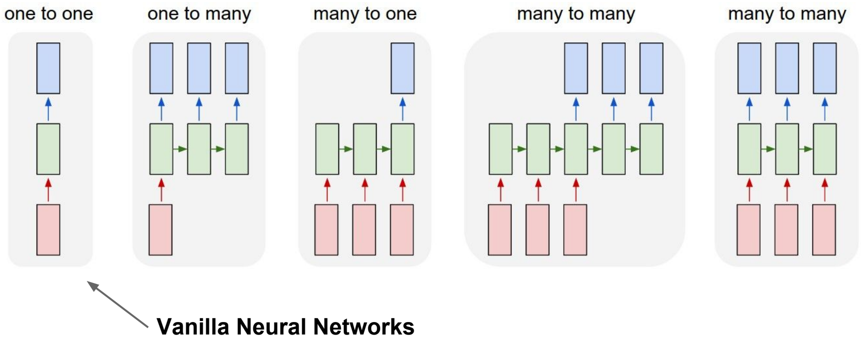
Conclusions

- RNNs allow a lot of flexibility in architecture design and have many applications

Recurrent Networks offer a lot of flexibility:



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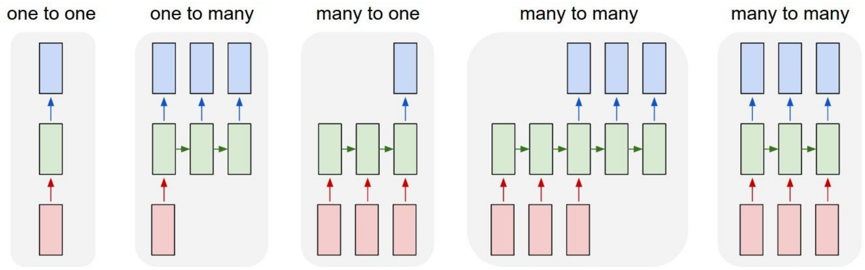

e.g. **Image Captioning**
image -> sequence of words

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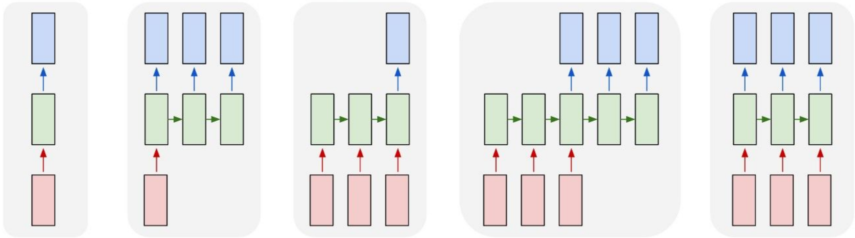
Recurrent Networks offer a lot of flexibility:

one to one | one to many | many to one | many to many | many to many



e.g. **Sentiment Classification**
sequence of words -> sentiment

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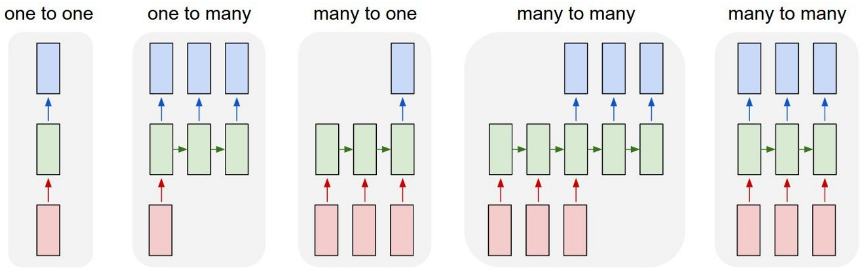

e.g. **Machine Translation**
seq of words -> seq of words

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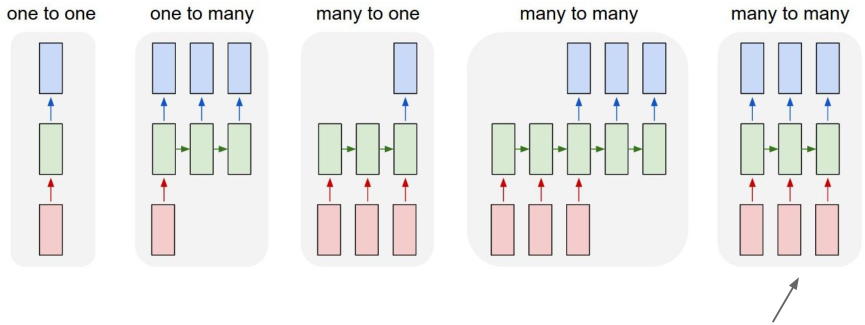
Recurrent Networks offer a lot of flexibility:



e.g. **Video classification on frame level**

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Conclusions

- RNNs allow a lot of flexibility in architecture design and have many applications
- Vanilla RNNs are simple but don't work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
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